GreatHearts®

Physics I

March 23-27

Time Allotment: 40 minutes per day

Student Name: ________________________________

Teacher Name: ________________________________

Physics I
March 23-27

Packet Overview

Date	Objective(s)	Page Number
Monday, March 23	1. Define rigid bodies and axis of rotation. 2. Compare radians to degree as measures of rotation. 3. Convert radians to degrees and degrees to radians.	2-3
Tuesday, March 24	1. Practice converting radian and degree measures. 2. Practice solving problems involving radians and degrees.	4-5
Wednesday, March 25	1. More practice converting degrees to radians and graphing radian and angle measures.	6 (plus Kuta Worksheet p. 1-2)
Thursday, March 26	1. Quiz on Radians and Degrees 2. Define angular velocity and acceleration. Compare quantities to linear velocity and acceleration.	7-10 (plus quiz)
Friday, March 27	1. Define radial and tangential accelaration. 2. Solve problems involving angular quantities.	11-12

Additional Notes: The guided worksheets in this packet will follow the textbook readings from Giancoli found at the end of the packet. The final page of this packet will contain an answer key for all Problems and answers to quiz questions.

Khan Academy is a great online resource for physics, though this packet does not require access to the Internet. The Physics videos can help with rotational motion concepts, while the algebra and geometry videos can help with the concept of radians.

Another great resource is a YouTube channel called "Doc Schuster". Dr. Schuster is a high school physics teacher in St. Louis who makes great video lectures with magic markers and paper. His playlist "AP Ch 10 – Rotational Motion and Energy" will cover most of we will in these packets.

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

Monday, March 23

Physics Unit: Rotational Motion
Lesson 1: Angular Quantities
Requirements: Read pages 194-195 in the textbook provided in the back of the packet and complete the worksheet below.

Unit Overview: Rotational Motion
In this new unit, we will be taking what we have already learned about linear velocity, acceleration, and momentum, and apply them to rotational cases. This will be different from our Chapter 5 unit on circular motion, because as you remember, objects in that chapter orbited in circles (think about the tennis ball on the string and the Moon orbiting around the Earth). In this chapter, we will be concerned with the rotation of the bodies themselves. These rotating bodies can be anything from a penny spinning on its side, you and your friends riding a Merry-Go-Round, a planet making its daily rotation, or an electron spinning. You should be getting excited! Towards the end of this chapter, we will get to see how the fundamentals of rotational motion we will learn leads to one of the most stunning demonstrations in all of mechanics. Stay tuned.

Lesson 1 Objective: Be able to do this by the end of this lesson.
1. Define rigid object, axis of rotation, and radian.
2. Convert radians to degrees, and degrees to radians.

Introduction to Lesson 1
The reading for Lesson 1 will be **pages 194-195** in the Giancoli text provided in this packet. Read these pages carefully, and then fill out the worksheet below.

1. Define a <u>rigid body</u> –

2. Define the <u>axis of rotation</u> –

In the space below, draw figure 8-1(a). Label all points and angles.

Now find the axis of rotation for the circle. Put your pencil on the point. Imagine the circle rotating around your pencil tip. That is the axis of rotation. Amazing! Draw an 'x' at this point.

3. How do we indicate how far an object has rotated?

4. A point *P* moves through an angle θ when it travels ____

____.

5. The mathematics of circular motion is much easier if we use ___________________________ for angular measure.

6. Define a <u>radian</u> –

Draw Figure 8-1(b) in the space below and label all parts.

7. Write Equation **(8-1a)** in the space below.

8. What will theta equal if the arc length is equal to the radius? ________ rad (always use this unit!)

9. Explain why an arc length equal to the circumference of a circle is $l = 2\pi$.

10. Show the steps demonstrating that $\theta = 2\pi$ rad in a complete circle.

11. In the space below, write out the question for Example 8-1: Bike Wheel on p. 195. Then try to solve it without looking. After you have tried, check your answer with the book's and then look at the solution if you still need help.

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Tuesday, March 24

Physics Unit: Rotational Motion
Lesson 2: Radians and Degrees Practice Problems
Requirements: Work all steps for Example 8-2 and do all end-of-chapter Questions and Problems listed in this lesson. End-of-chapter Questions and Problems are at the end of your packet.

Objective: Be able to do this by the end of this lesson.
1. Solve problems involving radian and degree measurements.

Introduction to Lesson 2
Yesterday, we learned what rotational motion means. We also learned the difference between radian and degree measures, and learned how to convert from one to the other. Today, we're going to put what we learned into practice with some excellent practice problems!

1. In the space below, write the question for Example 8-2: Birds of prey – in radians on p. 196. Then try to solve it without looking at the Approach or the Solution. If you need a hint, look at the Approach. Check your answer, and if you need more help, then you can look at the Solution.

Now, in the spaces provided, do the following: **Question 1 on p. 217, Problems 1-4 on p. 219.** Remember, always draw and label a diagram first. Then, write your given information, then any equations, then solve the problem. Finally, check your answers with a red pen with the key at the back of the packet.

Question 1)

Problems:
1)

2)

3)

4)

Wednesday, March 25

Physics Unit: Rotational Motion
Lesson 3: Radians and Degrees Practice Problems
Requirements: Do “Kuta Software” worksheet located on the next page.

Objective: Be able to do this by the end of this lesson.
1. Convert radians to degrees and degrees to radians.
2. Graph radian and degree measures.

Introduction to Lesson 3
Today we are going to continue with more practice, this time involving drawing angles on graphs and determining both their angle and radian measure. Look in your packet for a worksheet titled “Kuta Software – Infinite Algebra 2: Angles and Angle Measure.” Do problems 1-6 and 11-24. Then check your answers with the answer key, also located in your packet. Remember as always **attempt all the problems with a pencil before checking your answers. When you check your answers, use your red pen for corrections.** After you have checked your answers, you can try 7-10 as a challenge exercise. They deal with converting decimals of degrees into minutes and seconds. This is how astronomers measure the locations of celestial bodies.

Name________________________________

Angles and Angle Measure

Date________________ Period____

Convert each degree measure into radians and each radian measure into degrees.

1) $325°$

2) $60°$

3) $-\frac{4\pi}{3}$

4) $\frac{23\pi}{12}$

5) $570°$

6) $-315°$

Convert each decimal degree measure into degrees-minutes-seconds and each degrees-minutes-seconds into decimal degrees.

7) $128.77°$

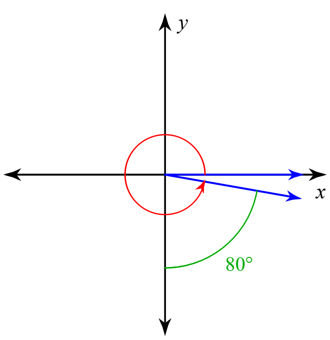
8) $232° 7' 57''$

9) $-154° 47' 42''$

10) $-0.9225°$

Find the measure of each angle.

11)



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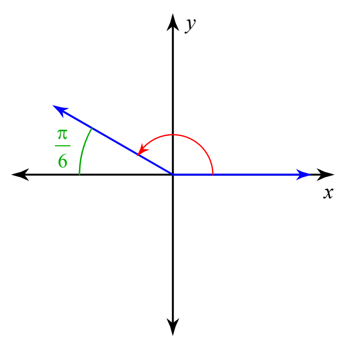

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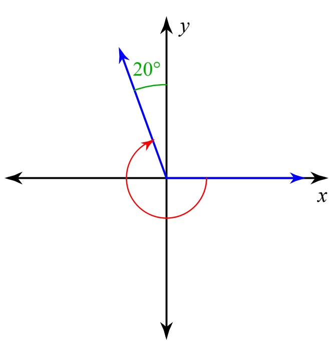

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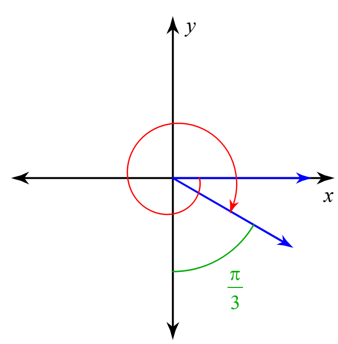

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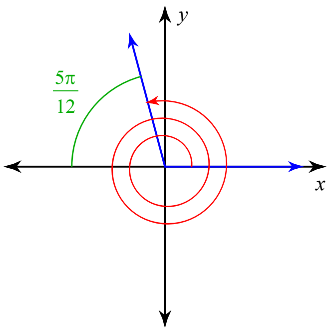

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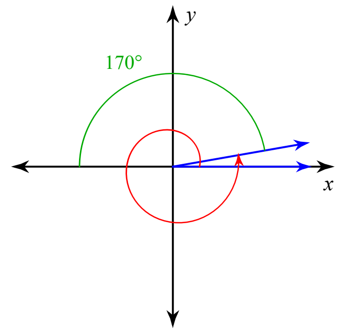
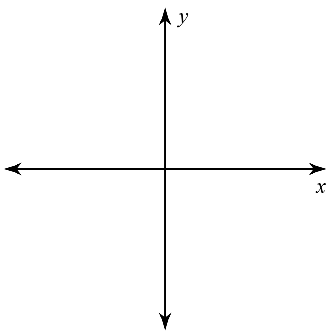
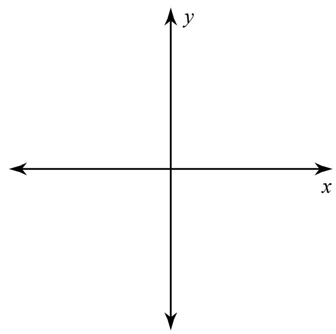


Draw an angle with the given measure in standard position.

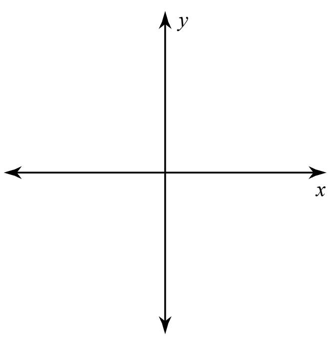
17) $280°$



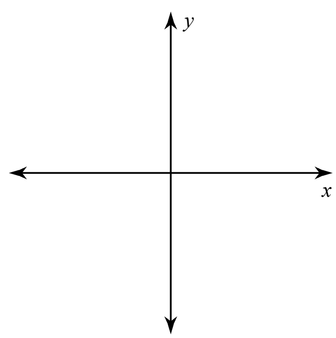
18) $710°$



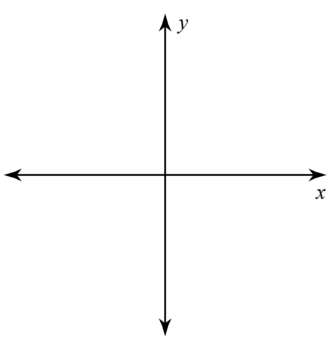
19) $-120°$



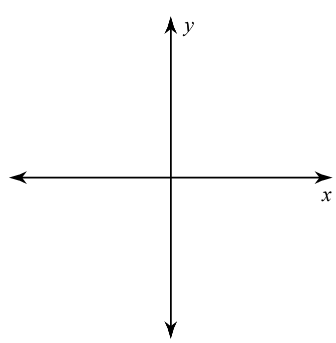
20) $\frac{11\pi}{6}$



21) $-\frac{10\pi}{3}$



22) $440°$



State the quadrant in which the terminal side of each angle lies.

23) $-509°$

24) $-\frac{5\pi}{6}$

Thursday, March 26

Physics Unit: Rotational Motion
Lesson 4: Quiz on Radians and Degrees, Angular Velocity, and Angular Acceleration
Requirements: Take Quiz on Radians and Degrees, read p. 196-197 in Giancoli, and complete the worksheet below.

Objective: Be able to do this by the end of this lesson.
1. Demonstrate mastery of radians and degrees on your quiz.
2. Define angular velocity and angular acceleration.
3. Compare those quantities to linear velocity and acceleration.

Physics I – Quiz on Radians and Degrees Name: ________________________

Convert each degree measure into radians and each radian measure into degrees. When you are finished, check your answers with the answer key in the back of the packet with your red pen.

1) $-\pi/6$

2) $-23\pi/6$

3) -30°

4) -930°

5) -210°

6) $\pi/4$

7) -160°

8) $-\pi/3$

9) $11\pi/6$

10) $17\pi/12$

11) 915°

12) $\pi/2$

Introduction to Lesson 4

Today we are going to learn about two new quantities, angular velocity and angular acceleration. These are very similar in name and concept to linear velocity and linear acceleration, which we have studied in previous chapters. Pay close attention to what is different, especially the use of radians as a measure of displacement!

1. Define average angular velocity (include Equation 8-2a in your definition) –

2. Define instantaneous angular velocity (include Equation 8-2b in your definition) –

3. What symbol do we use to for angular velocity? What units do we use?

4. True or false: Points closer to the center of a rotating object have a greater angular velocity. Explain.

5. Define average angular acceleration (include Equation 8-3a in your definition) –

6. Define instantaneous angular acceleration (include Equation 8-3b in your definition) –

7. What symbol do we use to for angular acceleration? What units do we use?

8. In the space below, draw and label Figure 8-4 on p. 197.

9. If we take a point P on the rotating circle in Figure 8-4, which direction will is linear velocity be pointing in relation to its circular path? What is its magnitude?

10. Show the steps required to derive Equation (8-4) on p. 197. Write out annotations to explain each step for full credit.

10. In the space below, draw and label Figure 8-5 on p. 197.

11. Use Figure 8-5 to explain why linear velocity increases for points on the circle further from the axis of rotation.

Friday, March 27

Physics Unit: Rotational Motion
Lesson 5: Angular Velocity and Angular Acceleration Continued
Requirements: Read p. 198-199 in Giancoli and complete the worksheet below.

Objective: Be able to do this by the end of this lesson.
1. Derive equation for tangential acceleration.
2. Derive the equation for radial acceleration.
3. Describe the difference between tangential and radial acceleration.

1. Show the steps to derive Equation (8-5) on p. 198. What quantity did you derive?

2. Show the steps to derive Equation (8-6) on p. 198. What quantity did you derive?

3. In your own words, what is the difference between tangential and radial acceleration? What quantity do you get when you add the two vectors together? Draw and label Figure 8-6 below to help illustrate your point.

4. Why is centripetal acceleration greater the farther you are from the axis of rotation?

5. In the space below, draw Table 8-1 at the bottom of p. 198.

6. In the space below, work all steps for Example 8-4 – Angular and linear velocities and accelerations on p. 199. Include sketches of Figures 8-4(a) and (b).

You too can experience rapid rotation—if your stomach can take the high angular velocity and centripetal acceleration of some of the faster amusement park rides. If not, try the slower merry-go-round or Ferris wheel. Rotating carnival rides have rotational KE as well as angular momentum.

CHAPTER 8

Rotational Motion

Until now, we have been concerned mainly with translational motion. We discussed the kinematics and dynamics of translational motion (the role of force), and the energy and momentum associated with it. In this Chapter we will deal with rotational motion. We will discuss the kinematics of rotational motion and then its dynamics (involving torque), as well as rotational kinetic energy and angular momentum (the rotational analog of linear momentum). We will find many analogies with translational motion, which will make our study easier. Our understanding of the world around us will be increased significantly—from rotating bicycle wheels and compact disks to amusement park rides, a spinning skater, the rotating Earth, and a centrifuge—and there may be a few surprises.

We will consider mainly the rotation of rigid objects. A **rigid object** is an object with a definite shape that doesn't change, so that the particles composing it stay in fixed positions relative to one another. Any real object is capable of vibrating or deforming when a force is exerted on it. But these effects are often very small, so the concept of an ideal rigid object is very useful as a good approximation.

8–1 Angular Quantities

We saw in Chapter 7 (Section 7–8) that the motion of a rigid object can be analyzed as the translational motion of the object's center of mass, plus rotational motion *about* its center of mass. We have already discussed translational motion in detail, so now we focus on purely rotational motion. By *purely rotational motion*, we mean that all points in the object move in circles, such as the point P in the rotating wheel of Fig. 8–1, and that the centers of these circles all lie on a line called the **axis of rotation**. In Fig. 8–1 the axis of rotation is perpendicular to the page and passes through point O.

Every point in an object rotating about a fixed axis moves in a circle (shown dashed in Fig. 8–1 for point P) whose center is on the axis and whose radius is r, the distance of that point from the axis of rotation. A straight line drawn from the axis to any point sweeps out the same angle θ in the same time.

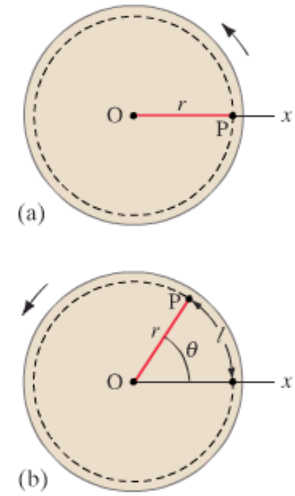


FIGURE 8–1 Looking at a wheel that is rotating counterclockwise about an axis through the wheel's center at O (axis perpendicular to the page). Each point, such as point P, moves in a circular path; l is the distance P travels as the wheel rotates through the angle θ.

To indicate the angular position of a rotating object, or how far it has rotated, we specify the angle θ of some particular line in the object (red in Fig. 8–1) with respect to a reference line, such as the x axis in Fig. 8–1. A point in the object, such as P in Fig. 8–1, moves through an angle θ when it travels the distance l measured along the circumference of its circular path. Angles are commonly measured in degrees, but the mathematics of circular motion is much simpler if we use the *radian* for angular measure. One **radian** (abbreviated rad) is defined as the angle subtended by an arc whose length is equal to the radius. For example, in Fig. 8–1b, point P is a distance r from the axis of rotation, and it has moved a distance l along the arc of a circle. The arc length l is said to "subtend" the angle θ. If $l = r$, then θ is exactly equal to 1 rad. In radians, any angle θ is given by

$$\theta = \frac{l}{r}, \qquad \textbf{(8–1a)}$$

θ in radians

where r is the radius of the circle, and l is the arc length subtended by the angle specified in radians. If $l = r$, then $\theta = 1$ rad.

1 rad: arc length = radius

The radian is dimensionless since it is the ratio of two lengths. Nonetheless when giving an angle in radians, we always mention rad to remind us it is not degrees. It is often useful to rewrite Eq. 8–1a in terms of arc length l:

$$l = r\theta. \qquad \textbf{(8–1b)}$$

Radians can be related to degrees in the following way. In a complete circle there are 360°, which must correspond to an arc length equal to the circumference of the circle, $l = 2\pi r$. Thus $\theta = l/r = 2\pi r/r = 2\pi\,\text{rad}$ in a complete circle, so

$$360° = 2\pi\,\text{rad}.$$

Conversion, degrees to rad

One radian is therefore $360°/2\pi \approx 360°/6.28 \approx 57.3°$. An object that makes one complete revolution (rev) has rotated through 360°, or 2π radians:

1 rad ≈ 57.3°

$$1\,\text{rev} = 360° = 2\pi\,\text{rad}.$$

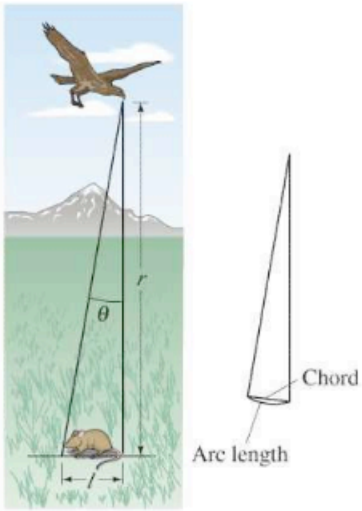
EXAMPLE 8–1 **Bike wheel.** A bike wheel rotates 4.50 revolutions. How many radians has it rotated?

APPROACH All we need is a straightforward conversion of units using

$$1\,\text{revolution} = 360° = 2\pi\,\text{rad} = 6.28\,\text{rad}.$$

SOLUTION

$$4.50\,\text{revolutions} = (4.50\,\text{rev})\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right) = 9.00\pi\,\text{rad} = 28.3\,\text{rad}.$$



(a) (b)

FIGURE 8–2 (a) Example 8–2. (b) For small angles, arc length and the chord length (straight line) are nearly equal.

EXAMPLE 8–2 **Birds of prey—in radians.** A particular bird's eye can just distinguish objects that subtend an angle no smaller than about 3×10^{-4} rad. (*a*) How many degrees is this? (*b*) How small an object can the bird just distinguish when flying at a height of 100 m (Fig. 8–2a)?

APPROACH For (*a*) we use the relation $360° = 2\pi$ rad. For (*b*) we use Eq. 8–1b, $l = r\theta$, to find the arc length.

SOLUTION (*a*) We convert 3×10^{-4} rad to degrees:

$$(3 \times 10^{-4}\,\text{rad})\left(\frac{360°}{2\pi\,\text{rad}}\right) = 0.017°.$$

(*b*) We use Eq. 8–1b, $l = r\theta$. For small angles, the arc length l and the chord length are approximately[†] the same (Fig. 8–2b). Since $r = 100$ m and $\theta = 3 \times 10^{-4}$ rad, we find

$$l = (100\,\text{m})(3 \times 10^{-4}\,\text{rad}) = 3 \times 10^{-2}\,\text{m} = 3\,\text{cm}.$$

A bird can distinguish a small mouse (about 3 cm long) from a height of 100 m. That is good eyesight.

NOTE Had the angle been given in degrees, we would first have had to convert it to radians to make this calculation. Equation 8–1 is valid *only* if the angle is specified in radians. Degrees (or revolutions) won't work.

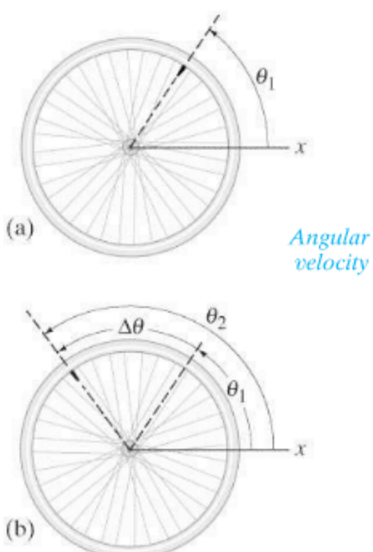
To describe rotational motion, we make use of angular quantities, such as angular velocity and angular acceleration. These are defined in analogy to the corresponding quantities in linear motion, and are chosen to describe the rotating object as a whole, so they are the same for each point in the rotating object. Each point in a rotating object may also have translational velocity and acceleration, but they have different values for different points in the object.

Angular displacement (rad)

When an object, such as the bicycle wheel in Fig. 8–3, rotates from some initial position, specified by θ_1, to some final position, θ_2, its *angular displacement* is

$$\Delta\theta = \theta_2 - \theta_1.$$

FIGURE 8–3 A wheel rotates from (a) initial position θ_1 to (b) final position θ_2. The angular displacement is $\Delta\theta = \theta_2 - \theta_1$.



The *angular velocity* (denoted by ω, the Greek lowercase letter omega) is defined in analogy with linear (translational) velocity that was discussed in Chapter 2. Instead of linear displacement, we use the angular displacement. Thus the **average angular velocity** is defined as

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}, \tag{8–2a}$$

where $\Delta\theta$ is the angle through which the object has rotated in the time interval Δt. We define the **instantaneous angular velocity** as the very small angle $\Delta\theta$, through which the object turns in the very short time interval Δt:

Angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}. \tag{8–2b}$$

Angular velocity is generally specified in radians per second (rad/s). Note that *all points in a rigid object rotate with the same angular velocity*, since every position in the object moves through the same angle in the same time interval.

An object such as the wheel in Fig. 8–3 can rotate about a fixed axis either clockwise or counterclockwise. The direction can be specified with a + or − sign, just as we did in Chapter 2 for linear motion along the $+x$ or $-x$ axis. The usual convention is to choose the angular displacement $\Delta\theta$ and angular velocity ω as positive when the wheel rotates counterclockwise. If the rotation is clockwise, then θ would decrease, so $\Delta\theta$ and ω would be negative.[‡]

[†]Even for an angle as large as 15°, the error in making this estimate is only 1%, but for larger angles the error increases rapidly.

[‡]The vector nature of angular velocity and other angular quantities is discussed in Section 8–9 (optional).

Angular acceleration (denoted by α, the Greek lowercase letter alpha), in analogy to linear acceleration, is defined as the change in angular velocity divided by the time required to make this change. The **average angular acceleration** is defined as

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t}, \tag{8–3a}$$

where ω_1 is the angular velocity initially, and ω_2 is the angular velocity after a time interval Δt. **Instantaneous angular acceleration** is defined in the usual way as the limit of this ratio as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}. \tag{8–3b}$$

Angular acceleration

Since ω is the same for all points of a rotating object, Eq. 8–3 tells us that α also will be the same for all points. Thus, ω and α are properties of the rotating object as a whole. With ω measured in radians per second and t in seconds, α will be expressed as radians per second squared $(\mathrm{rad/s^2})$.

Each point or particle of a rotating rigid object has, at any moment, a linear velocity v and a linear acceleration a. We can relate the linear quantities at each point, v and a, to the angular quantities of the rotating object, ω and α. Consider a point P located a distance r from the axis of rotation, as in Fig. 8–4. If the object rotates with angular velocity ω, any point will have a linear velocity whose direction is tangent to its circular path. The magnitude of that point's linear velocity is $v = \Delta l/\Delta t$. From Eq. 8–1b, a change in rotation angle $\Delta\theta$ (in radians) is related to the linear distance traveled by $\Delta l = r\,\Delta\theta$. Hence

$$v = \frac{\Delta l}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

or

$$v = r\omega. \tag{8–4}$$

Linear and angular velocity related

Thus, although ω is the same for every point in the rotating object at any instant, the linear velocity v is greater for points farther from the axis (Fig. 8–5). Note that Eq. 8–4 is valid both instantaneously and on the average.

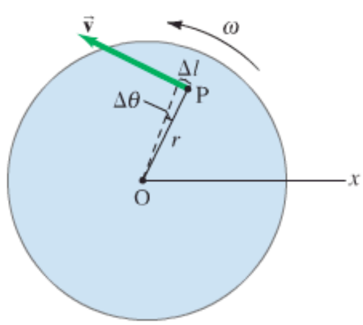


FIGURE 8–4 A point P on a rotating wheel has a linear velocity $\vec{\mathbf{v}}$ at any moment.

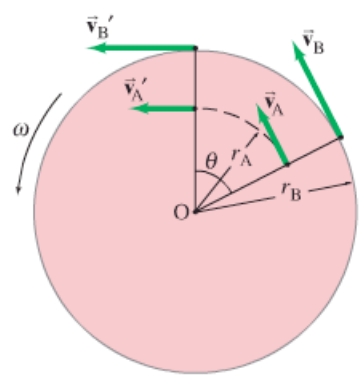


FIGURE 8–5 A wheel rotating uniformly counterclockwise. Two points on the wheel, at distances r_A and r_B from the center, have the same angular velocity ω because they travel through the same angle θ in the same time interval. But the two points have different linear velocities because they travel different distances in the same time interval. Since $r_B > r_A$, then $v_B > v_A$ $(v = r\omega)$.

CONCEPTUAL EXAMPLE 8–3 **Is the lion faster than the horse?** On a rotating carousel or merry-go-round, one child sits on a horse near the outer edge and another child sits on a lion halfway out from the center. (*a*) Which child has the greater linear velocity? (*b*) Which child has the greater angular velocity?

RESPONSE (*a*) The *linear* velocity is the distance traveled divided by the time interval. In one rotation the child on the outer edge travels a longer distance than the child near the center, but the time interval is the same for both. Thus the child at the outer edge, on the horse, has the greater linear velocity.
(*b*) The *angular* velocity is the angle of rotation divided by the time interval. In one rotation both children rotate through the same angle ($360° = 2\pi$ radians). The two children have the same angular velocity.

If the angular velocity of a rotating object changes, the object as a whole—and each point in it—has an angular acceleration. Each point also has a linear acceleration whose direction is tangent to that point's circular path. We use Eq. 8–4 ($v = r\omega$) to show that the angular acceleration α is related to the tangential linear acceleration a_{tan} of a point in the rotating object by

$$a_{\mathrm{tan}} = \frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

or

Tangential acceleration

$$a_{\mathrm{tan}} = r\alpha. \qquad \textbf{(8–5)}$$

In this equation, r is the radius of the circle in which the particle is moving, and the subscript "tan" in a_{tan} stands for "tangential."

The total linear acceleration of a point is the vector sum of two components:

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\mathrm{tan}} + \vec{\mathbf{a}}_{\mathrm{R}},$$

where the radial† component, $\vec{\mathbf{a}}_{\mathrm{R}}$, is the radial or "centripetal" acceleration and its direction is toward the center of the point's circular path; see Fig. 8–6. We saw in Chapter 5 (Eq. 5–1) that $a_{\mathrm{R}} = v^2/r$, and we can rewrite this in terms of ω using Eq. 8–4:

Centripetal (or radial) acceleration

$$a_{\mathrm{R}} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r. \qquad \textbf{(8–6)}$$

Thus the centripetal acceleration is greater the farther you are from the axis of rotation: the children farthest out on a carousel feel the greatest acceleration. Equations 8–4, 8–5, and 8–6 relate the angular quantities describing the rotation of an object to the linear quantities for each point of the object. Table 8–1 summarizes these relationships.

FIGURE 8–6 On a rotating wheel whose angular speed is increasing, a point P has both tangential and radial (centripetal) components of linear acceleration. (See also Chapter 5.)

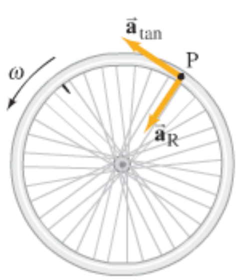


TABLE 8–1 Linear and Rotational Quantities

Linear	Type	Rotational	Relation
x	displacement	θ	$x = r\theta$
v	velocity	ω	$v = r\omega$
a_{tan}	acceleration	α	$a_{\mathrm{tan}} = r\alpha$

† "Radial" means along the radius—that is, toward or away from the center or axis.

EXAMPLE 8–4 **Angular and linear velocities and accelerations.** A carousel is initially at rest. At $t = 0$ it is given a constant angular acceleration $\alpha = 0.060\ \text{rad/s}^2$, which increases its angular velocity for 8.0 s. At $t = 8.0\ \text{s}$, determine the following quantities: (*a*) the angular velocity of the carousel; (*b*) the linear velocity of a child (Fig. 8–7a) located 2.5 m from the center, point P in Fig. 8–7b; (*c*) the tangential (linear) acceleration of that child; (*d*) the centripetal acceleration of the child; and (*e*) the total linear acceleration of the child.

APPROACH The angular acceleration α is constant, so we can use Eq. 8–3a to solve for ω after a time $t = 8.0\ \text{s}$. With this ω and the given α, we determine the other quantities using the relations we just developed, Eqs. 8–4, 8–5, and 8–6.

SOLUTION (*a*) Equation 8–3a tells us

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t}.$$

We are given $\Delta t = 8.0\ \text{s}$, $\bar{\alpha} = 0.060\ \text{rad/s}^2$, and $\omega_1 = 0$. Solving for ω_2, we get

$$\begin{aligned}\omega_2 &= \omega_1 + \bar{\alpha}\,\Delta t \\ &= 0 + (0.060\ \text{rad/s}^2)(8.0\ \text{s}) = 0.48\ \text{rad/s}.\end{aligned}$$

During the 8.0-s interval, the carousel has accelerated from $\omega_1 = 0$ (rest) to $\omega_2 = 0.48\ \text{rad/s}$.

(*b*) The linear velocity of the child with $r = 2.5\ \text{m}$ at time $t = 8.0\ \text{s}$ is found using Eq. 8–4:

$$v = r\omega = (2.5\ \text{m})(0.48\ \text{rad/s}) = 1.2\ \text{m/s}.$$

Note that the "rad" has been dropped here because it is dimensionless (and only a reminder)—it is a ratio of two distances, Eq. 8–1b.

(*c*) The child's tangential acceleration is given by Eq. 8–5:

$$a_{\tan} = r\alpha = (2.5\ \text{m})(0.060\ \text{rad/s}^2) = 0.15\ \text{m/s}^2,$$

and it is the same throughout the 8.0-s acceleration interval.

(*d*) The child's centripetal acceleration at $t = 8.0\ \text{s}$ is given by Eq. 8–6:

$$a_R = \frac{v^2}{r} = \frac{(1.2\ \text{m/s})^2}{(2.5\ \text{m})} = 0.58\ \text{m/s}^2.$$

(*e*) The two components of linear acceleration calculated in parts (*c*) and (*d*) are perpendicular to each other. Thus the total linear acceleration at $t = 8.0\ \text{s}$ has magnitude

$$\begin{aligned}a &= \sqrt{a_{\tan}^2 + a_R^2} \\ &= \sqrt{(0.15\ \text{m/s}^2)^2 + (0.58\ \text{m/s}^2)^2} = 0.60\ \text{m/s}^2.\end{aligned}$$

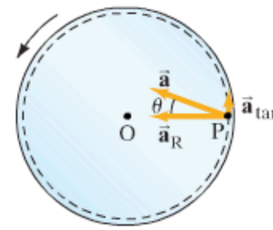
Its direction (Fig. 8–7b) is

$$\theta = \tan^{-1}\left(\frac{a_{\tan}}{a_R}\right) = \tan^{-1}\left(\frac{0.15\ \text{m/s}^2}{0.58\ \text{m/s}^2}\right) = 0.25\ \text{rad},$$

so $\theta \approx 15°$.

NOTE The linear acceleration is mostly centripetal, keeping the child moving in a circle with the carousel. The tangential component that speeds up the motion is smaller.

(a)



(b)

FIGURE 8–7 Example 8–4. The total acceleration vector $\vec{a} = \vec{a}_{\tan} + \vec{a}_R$, at $t = 8.0\ \text{s}$.

Summary

When a rigid object rotates about a fixed axis, each point of the object moves in a circular path. Lines drawn perpendicularly from the rotation axis to various points in the object all sweep out the same angle θ in any given time interval.

Angles are conveniently measured in **radians**, where one radian is the angle subtended by an arc whose length is equal to the radius, or

$$2\pi \text{ rad} = 360^\circ$$
$$1 \text{ rad} \approx 57.3^\circ.$$

Angular velocity, ω, is defined as the rate of change of angular position:

$$\omega = \frac{\Delta\theta}{\Delta t}. \qquad \textbf{(8–2)}$$

All parts of a rigid object rotating about a fixed axis have the same angular velocity at any instant.

Angular acceleration, α, is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}. \qquad \textbf{(8–3)}$$

The linear velocity v and acceleration a of a point fixed at a distance r from the axis of rotation are related to ω and α by

$$v = r\omega, \qquad \textbf{(8–4)}$$
$$a_{\tan} = r\alpha, \qquad \textbf{(8–5)}$$
$$a_R = \omega^2 r, \qquad \textbf{(8–6)}$$

where $a_{\tan}$ and a_R are the tangential and radial (centripetal) components of the linear acceleration, respectively.

The frequency f is related to ω by

$$\omega = 2\pi f, \qquad \textbf{(8–7)}$$

and to the period T by

$$T = 1/f. \qquad \textbf{(8–8)}$$

The equations describing uniformly accelerated rotational motion (α = constant) have the same form as for uniformly accelerated linear motion:

$$\omega = \omega_0 + \alpha t, \qquad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta, \qquad \bar{\omega} = \frac{\omega + \omega_0}{2}. \qquad \textbf{(8–9)}$$

The dynamics of rotation is analogous to the dynamics of linear motion. Force is replaced by **torque** τ, which is defined as the product of force times lever arm (perpendicular distance from the line of action of the force to the axis of rotation):

$$\tau = rF\sin\theta = r_\perp F = rF_\perp. \qquad \textbf{(8–10)}$$

Mass is replaced by **moment of inertia** I, which depends not only on the mass of the object, but also on how the mass is distributed about the axis of rotation. Linear acceleration is replaced by angular acceleration. The rotational equivalent of Newton's second law is then

$$\Sigma\tau = I\alpha. \qquad \textbf{(8–14)}$$

The **rotational kinetic energy** of an object rotating about a fixed axis with angular velocity ω is

$$\text{KE} = \tfrac{1}{2}I\omega^2. \qquad \textbf{(8–15)}$$

For an object both translating and rotating, the total kinetic energy is the sum of the translational kinetic energy of the object's center of mass plus the rotational kinetic energy of the object about its center of mass:

$$\text{KE} = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 \qquad \textbf{(8–16)}$$

as long as the rotation axis is fixed in direction.

The **angular momentum** L of an object about a fixed rotation axis is given by

$$L = I\omega. \qquad \textbf{(8–18)}$$

Newton's second law, in terms of angular momentum, is

$$\Sigma\tau = \frac{\Delta L}{\Delta t}. \qquad \textbf{(8–19)}$$

If the net torque on the object is zero, $\Delta L/\Delta t = 0$, so L = constant. This is the **law of conservation of angular momentum** for a rotating object.

The following Table summarizes angular (or rotational) quantities, comparing them to their translational analogs.

Translation	Rotation	Connection
x	θ	$x = r\theta$
v	ω	$v = r\omega$
a	α	$a = r\alpha$
m	I	$I = \Sigma mr^2$
F	τ	$\tau = rF\sin\theta$
$\text{KE} = \frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	
$p = mv$	$L = I\omega$	
$W = Fd$	$W = \tau\theta$	
$\Sigma F = ma$	$\Sigma\tau = I\alpha$	
$\Sigma F = \frac{\Delta p}{\Delta t}$	$\Sigma\tau = \frac{\Delta L}{\Delta t}$	

Questions

1. A bicycle odometer (which measures distance traveled) is attached near the wheel hub and is designed for 27-inch wheels. What happens if you use it on a bicycle with 24-inch wheels?
2. Suppose a disk rotates at constant angular velocity. Does a point on the rim have radial and/or tangential acceleration? If the disk's angular velocity increases uniformly, does the point have radial and/or tangential acceleration? For which cases would the magnitude of either component of linear acceleration change?
3. Could a nonrigid body be described by a single value of the angular velocity ω? Explain.
4. Can a small force ever exert a greater torque than a larger force? Explain.

Problems

8–1 Angular Quantities

1. (I) Express the following angles in radians: (*a*) 30°, (*b*) 57°, (*c*) 90°, (*d*) 360°, and (*e*) 420°. Give as numerical values and as fractions of π.

2. (I) Eclipses happen on Earth because of an amazing coincidence. Calculate, using the information inside the Front Cover, the angular diameters (in radians) of the Sun and the Moon, as seen on Earth.

3. (I) A laser beam is directed at the Moon, 380,000 km from Earth. The beam diverges at an angle θ (Fig. 8–37) of 1.4×10^{-5} rad. What diameter spot will it make on the Moon?

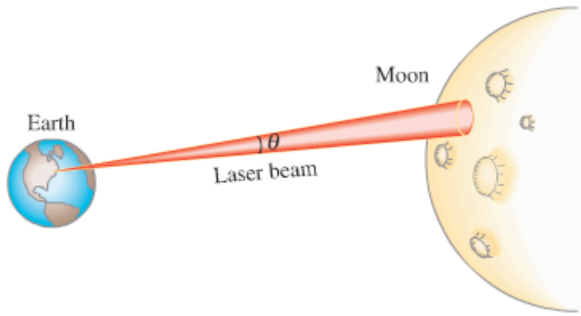


FIGURE 8–37 Problem 3.

4. (I) The blades in a blender rotate at a rate of 6500 rpm. When the motor is turned off during operation, the blades slow to rest in 3.0 s. What is the angular acceleration as the blades slow down?

5. (II) A child rolls a ball on a level floor 3.5 m to another child. If the ball makes 15.0 revolutions, what is its diameter?

6. (II) A bicycle with tires 68 cm in diameter travels 8.0 km. How many revolutions do the wheels make?

7. (II) (*a*) A grinding wheel 0.35 m in diameter rotates at 2500 rpm. Calculate its angular velocity in rad/s. (*b*) What are the linear speed and acceleration of a point on the edge of the grinding wheel?

8. (II) A rotating merry-go-round makes one complete revolution in 4.0 s (Fig. 8–38). (*a*) What is the linear speed of a child seated 1.2 m from the center? (*b*) What is her acceleration (give components)?

FIGURE 8–38 Problem 8.

9. (II) Calculate the angular velocity of the Earth (*a*) in its orbit around the Sun, and (*b*) about its axis.

10. (II) What is the linear speed of a point (*a*) on the equator, (*b*) on the Arctic Circle (latitude 66.5° N), and (*c*) at a latitude of 45.0° N, due to the Earth's rotation?

11. (II) How fast (in rpm) must a centrifuge rotate if a particle 7.0 cm from the axis of rotation is to experience an acceleration of 100,000 *g*'s?

12. (II) A 70-cm-diameter wheel accelerates uniformly about its center from 130 rpm to 280 rpm in 4.0 s. Determine (*a*) its angular acceleration, and (*b*) the radial and tangential components of the linear acceleration of a point on the edge of the wheel 2.0 s after it has started accelerating.

13. (II) A turntable of radius R_1 is turned by a circular rubber roller of radius R_2 in contact with it at their outer edges. What is the ratio of their angular velocities, ω_1/ω_2?

14. (III) In traveling to the Moon, astronauts aboard the *Apollo* spacecraft put themselves into a slow rotation to distribute the Sun's energy evenly. At the start of their trip, they accelerated from no rotation to 1.0 revolution every minute during a 12-min time interval. The spacecraft can be thought of as a cylinder with a diameter of 8.5 m. Determine (*a*) the angular acceleration, and (*b*) the radial and tangential components of the linear acceleration of a point on the skin of the ship 5.0 min after it started this acceleration.

8–2 and 8–3 Constant Angular Acceleration; Rolling

15. (I) A centrifuge accelerates uniformly from rest to 15,000 rpm in 220 s. Through how many revolutions did it turn in this time?

16. (I) An automobile engine slows down from 4500 rpm to 1200 rpm in 2.5 s. Calculate (*a*) its angular acceleration, assumed constant, and (*b*) the total number of revolutions the engine makes in this time.

17. (I) Pilots can be tested for the stresses of flying high-speed jets in a whirling "human centrifuge," which takes 1.0 min to turn through 20 complete revolutions before reaching its final speed. (*a*) What was its angular acceleration (assumed constant), and (*b*) what was its final angular speed in rpm?

18. (II) A wheel 33 cm in diameter accelerates uniformly from 240 rpm to 360 rpm in 6.5 s. How far will a point on the edge of the wheel have traveled in this time?

19. (II) A cooling fan is turned off when it is running at 850 rev/min. It turns 1500 revolutions before it comes to a stop. (*a*) What was the fan's angular acceleration, assumed constant? (*b*) How long did it take the fan to come to a complete stop?

20. (II) A small rubber wheel is used to drive a large pottery wheel, and they are mounted so that their circular edges touch. The small wheel has a radius of 2.0 cm and accelerates at the rate of $7.2\ \text{rad/s}^2$, and it is in contact with the pottery wheel (radius 25.0 cm) without slipping. Calculate (*a*) the angular acceleration of the pottery wheel, and (*b*) the time it takes the pottery wheel to reach its required speed of 65 rpm.

Physics I Packet 3-23 to 3-27 – Answer Key

Tuesday, March 24

Question 1: 24/27

Problems:

1) a) 0.52 rad b) 0.994 rad c) 1.57 rad d) 6.28 rad e) 7.33 rad

2) $\theta_{sun} = 9.3 \times 10^{-3}$ rad; $\theta_{moon} = 9.06 \times 10^{-3}$ rad

3) 5.3×10^{3} m

4) -227 rad/s^2

Quiz on Radians and Degrees

1) -30°

2) -690°

3) –π/6

4) -31π /6

5) -7 π/6

6) 45°

7) -8 π/9

8) -60°

9) 330°

10) 255°

11) 61 π/12

12) 90°

Kuta Software - Infinite Algebra 2 Name____________________

Angles and Angle Measure

Date________________ Period____

Convert each degree measure into radians and each radian measure into degrees.

1) $325°$ $\frac{65\pi}{36}$

2) $60°$ $\frac{\pi}{3}$

3) $-\frac{4\pi}{3}$

$-240°$

4) $\frac{23\pi}{12}$

$345°$

5) $570°$ $\frac{19\pi}{6}$

6) $-315°$ $-\frac{7\pi}{4}$

Convert each decimal degree measure into degrees-minutes-seconds and each degrees-minutes-seconds into decimal degrees.

7) 128.77°

128° 46' 12"

8) 232° 7' 57"

232.1325°

9) −154° 47' 42"

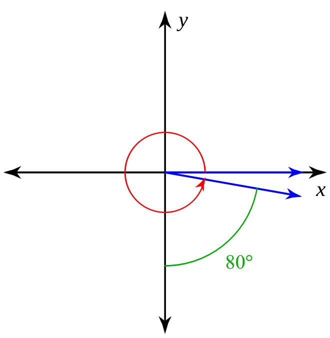
−154.795°

10) −0.9225°

−0° 55' 21"

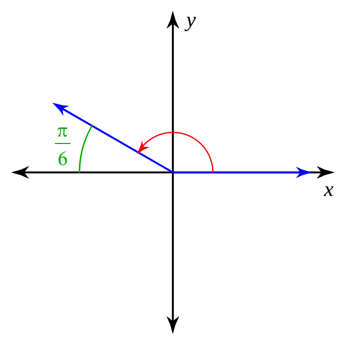
Find the measure of each angle.

11)

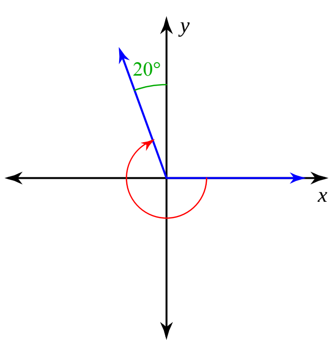


350°

12) $\frac{5\pi}{6}$

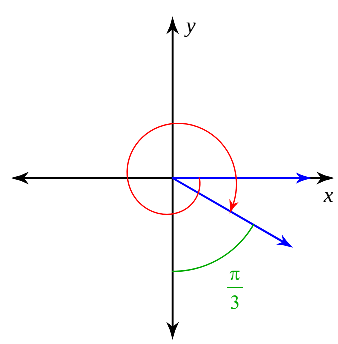


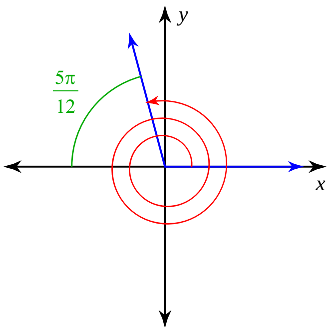
13)



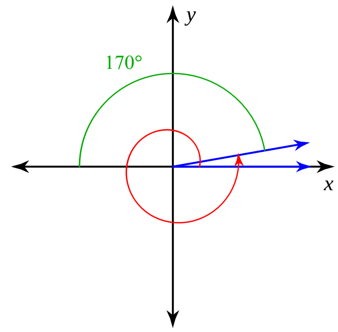
−250°

14) $-\frac{13\pi}{6}$



15) 

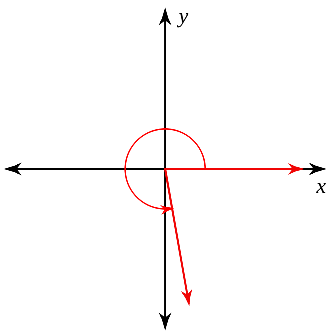
$\frac{55\pi}{12}$

16) 

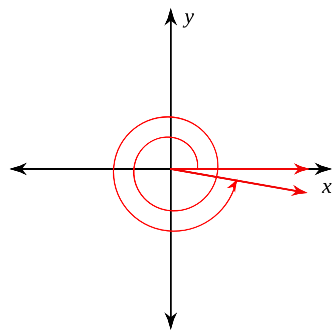
370°

Draw an angle with the given measure in standard position.

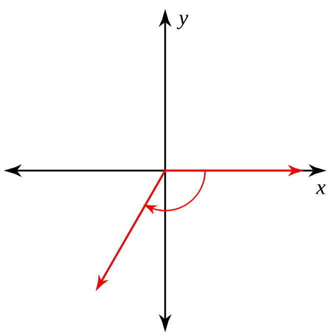
17) $280°$



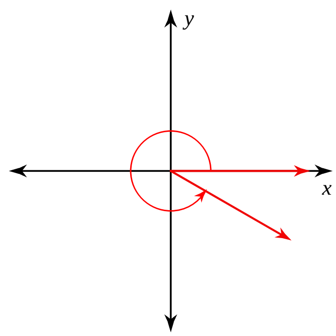
18) $710°$



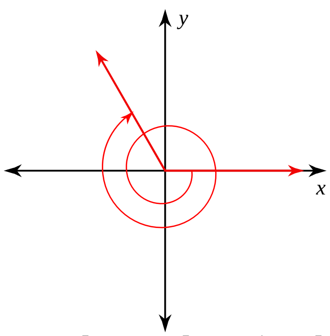
19) $-120°$



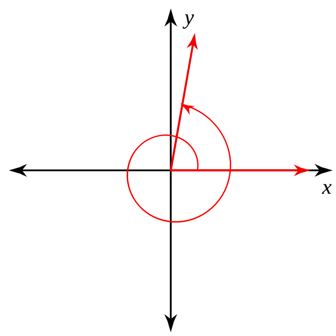
20) $\frac{11\pi}{6}$



21) $-\frac{10\pi}{3}$



22) $440°$



State the quadrant in which the terminal side of each angle lies.

23) $-509°$ III

24) $-\frac{5\pi}{6}$ III