GreatHearts®

Pre-Calculus: Week of March 23-27

Time Allotment: 40 minutes per day

Student Name: ________________________________

Teacher Name: Mrs. Melisa Walters

Pre-Calculus: Chapter 9 Systems of Equations and Inequalities
March 23-27

Packet Overview

Date	Objective(s)	Page Number
Monday, March 23	Recognize that we are dealing with more than one variable and likely more than one equation. Conclude the type of solution for the system of equations: a unique solution, infinitely many, or no solution.	2
Tuesday, March 24	Construct one linear equation to equal y and substitute it into the other equation.	5
Wednesday, March 25	Interchange any two equations of the system and/or replace any equation in the system by the sum of that equation and a nonzero multiple of any other equation in the system.	6
Thursday, March 26	Identifying Inconsistent Systems of Equations Containing Two Variables	10
Friday, March 27	Express the solution of a system of Dependent Equations containing two variables.	12

Additional Notes: We miss you all very much!!!

Any questions? Email me directly: melisa.walters@greatheartsnorthernoaks.org

Each lesson will end with a set of math problems pertaining to that particular lesson of the day.

Please create an "Exercise Packet" which is to include all of your work and completion of these daily exercises. Each day is to have a title with the date followed by the name of the lesson. Please include a title page and staple all the completed exercises. Answer key is on page 13. **At a later point, we will ask you to turn your exercise packet. Do not worry right now about whether that will be online or in person, simply do the problem set as I instruct with the proper titles and labels.**

Thank you for your hard work students. I appreciate all of you. Have a great day!

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

____

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

____

GreatHearts

Monday, March 23

Pre-Calculus Unit: Chapter 9
Lesson 1: Solve Linear Equations

Objective: Conclude the type of solution for the system of equations: a unique solution, infinitely many, or no solution.

NOTE: I will be providing some examples specific to this lesson. Please highlight the key points as you read through this entire lesson or take notes in the margin. At the end of the lesson, the list of questions in the exercise are to be completed to ensure mastery.

Preparing for this section:
Before getting started, let's review linear equations. Linear equations are equations such as

$3x + 12 = 0$, $\frac{3}{4}x - 5 = 0$, and $0.62x - 0.3 = 0$

A **linear equation** in one variable is equivalent to an equation of the form:

$ax + b = 0$ where a and b are real numbers and a cannot equal 0.

Linear equations in two variables:
If a, b, and r are real numbers (and if a and b are not both equal to 0) then $ax + by = r$ is called a linear equation in two variables. (The "two variables" are the x and the y.)

The numbers a and b are called the coefficients of the equation $ax+by = r$. The number r is called the constant of the equation $ax + by = r$.

Examples. $10x - 3y = 5$ and $-2x - 4y = 7$ are linear equations in two variables.

Solutions of equations

A *solution* of **a** linear equation in two variables $ax+by = r$ is a specific point such that when the x-coordinate of the point is multiplied by a, and the y-coordinate of the point is multiplied by b, and those two numbers are added together, the answer equals r.

Systems of linear equations

Rather than asking for the set of solutions of a single (1) linear equation in two variables, we could take **two different linear equations in two variables** and ask for all those points that are solutions to *both* of the linear equations.

For example, the point $x = 4$ and $y = 1$ is a solution of both of the equations:

$x + y = 5$ and $x - y = 3$.

If you have more than one linear equation, it's called **a system of linear equations**, so that

$$x + y = 5$$
$$x - y = 3$$

is an example of a system of two linear equations in two variables. There are two equations, and each equation has the same two variables: x and y.

A *solution of a system* of equations is a point that is a solution of each of the equations in the system.

Example. Consider the following system of linear equations in two variables.

$$2x + y = 15$$
$$3x - y = 5$$

The solution to a system of linear equations in two variables is any ordered pair that satisfies each equation independently. In this example above, the ordered pair (4, 7) is the solution to the system of linear equations. We can verify the solution by substituting the values $x = 4$ and $y = 7$ into each equation to see if the ordered pair satisfies both equations. Shortly we will investigate methods of finding such a solution if it exists.

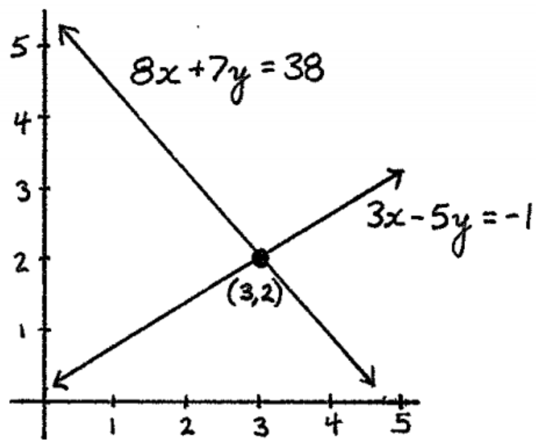
$$2(4) + (7) = 15 \text{ True}$$
$$3(4) - (7) = 5 \text{ True}$$

Almost all of the time, two different lines will intersect in a single point, so in these cases, there will only be one point that is a solution to both equations. Such a point is called the *unique solution* of the system of linear equations.

Example. Let's take a second look at the system of equations

$$8x + 7y = 38$$
$$3x - 5y = -1$$

The first equation in this system, $8x + 7y = 38$, corresponds to a line that has slope -8/7.
The second equation in this system, $3x-5y = -1$, is represented by a line that has slope 3/5. Since the two slopes are not equal, the lines have to intersect in exactly one point. That one point will be the unique solution. As we've seen before, x = 3 and y = 2 is a solution of this system. It is the unique solution.



Example. The system

$$5x + 2y = 4$$
$$2x + y = 11$$

has a unique solution. It's x = 2 and y = 7.

It's straightforward to check that x = 2 and y = 7 is a solution of the system. That it's the only solution follows from the fact that the slope of the line $5x + 2y = 4$ is different from slope of the line $2x + y = 11$. Those two slopes are 5 2 and 2 respectively.

No solutions

If you reach into a hat and pull out two different linear equations in two variables, it's highly unlikely that the two corresponding lines would have exactly the same slope. But if they did have the same slope, then there would not be a solution of the system of two linear equations since no point would lie on both of the parallel lines.

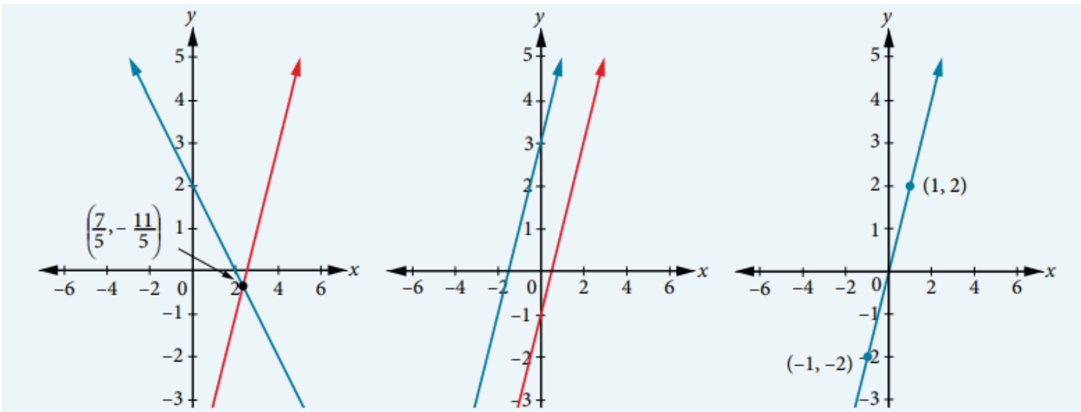
Example. The system

$$x - 2y = -4$$
$$-3x + 6y = 0$$

does not have a solution. That's because each of the two lines has the same slope, 1/2, so the lines don't intersect.

In summary, there are three types of systems of linear equations in two variables, and three types of solutions:

1. If the lines intersect, then the system of equations has one solution, given by the point of intersection. The system is **consistent** and the equations are **independent**.
2. If the lines are parallel, then the system of equations has no solution, because the lines never intersect. The system is **inconsistent**.
3. If the lines are **coincident**, then the system of equations has infinitely many solutions, represented by the totality of points on the line. The system is **consistent** and the equations are **dependent**.



Directions for completing the exercises:

1. Create a packet for completed exercises. I want you staple together several papers with a title page "Exercises for Pre-Calculus"
2. At the top of each exercise completion page, title as such: Exercises for Monday, March 23rd solving linear equations.
3. After you finish the entire exercise, please check your answers with the answer key at the end of the packet and attempt correction. (last page include answer key)

Exercises for Monday, March 23, 2020:

1.) What are the coefficients of the equation $2x - 5y = -23$?
2.) What is the constant of the equation $2x - 5y = -23$?
3.) Is $x = -4$ and $y = 3$ a solution of the equation $2x - 5y = -23$?
4.) What are the coefficients of the equation $-7x + 6y = 15$?
5.) What is the constant of the equation $-7x + 6y = 15$?
6.) Is $x = 3$ and $y = -10$ a solution of the equation $-7x + 6y = 15$?
7.) Is $x = 1$ and $y = 0$ a solution of the system:

$$x + y = 1$$
$$2x + 3y = 3$$

8.) Is $x = -1$ and $y = 3$ a solution of the system

$$7x + 2y = -1$$
$$5x - 3y = -14$$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Tuesday, March 24

Pre-Calculus: chapter 9
Lesson 2: solve system of equations by substitution

Objective: Solve systems of equations by substitution method
Construct one linear equation to equal y and substitute it into the other equation.

Lesson 2
Yesterday, we learned about the different solutions to the system of equations. Today we will learn the method of substitution to solve the systems of equations.

Solving a linear system in two variables by graphing works well when the solution consists of integer values, but if our solution contains decimals or fractions, it is not the most precise method. We will consider two more methods of solving a system of linear equations that are more precise than graphing. One such method is solving a system of equations by the **substitution method**, in which we solve one of the equations for one variable and then substitute the result into the second equation to solve for the second variable. Recall that we can solve for only one variable at a time, which is the reason the substitution method is both valuable and practical.

Given a system of two equations in two variables, solve using the substitution method.

1. Solve one of the two equations for one of the variables in terms of the other.
2. Substitute the expression for this variable into the second equation, then solve for the remaining variable.
3. Substitute that solution into either of the original equations to find the value of the first variable. If possible, write the solution as an ordered pair.
4. Check the solution in both equations.

Example: Solving a System of Equations in Two Variables by method Substitution

Solve the following system of equations by substitution.

$$-x + y = -5$$
$$2x - 5y = 1$$

Solution: First, we will solve the first equation for y.

$$-x + y = -5$$
$$y = x - 5$$

Now we can substitute the expression $x - 5$ for y in the second equation.

$$2x - 5y = 1$$
$$2x - 5(x - 5) = 1$$
$$2x - 5x + 25 = 1$$
$$-3x = -24$$
$$x = 8$$

Now, we substitute $x = 8$ into the first equation and solve for y.

$$-(8) + y = -5 \quad y = 3$$

Our solution is (8, 3).

Check the solution by substituting (8, 3) into both equations.
$-x + y = -5$
$-(8) + (3) = -5$ True

$2x - 5y = 1$
$2(8) - 5(3) = 1$ True

Question: Can the substitution method be used to solve any linear system in two variables?
Answer: Yes, but the method works best if one of the equations contains a coefficient of 1 or −1 so that we do not have to deal with fractions.

Exercises for March 24, 2020: Solve using the substitution method
For the following exercises, solve each system by substitution:

1. $x + 3y = 5$
   $2x + 3y = 4$

2. $4x + 2y = -10$
   $3x + 9y = 0$

3. $3x + 5y = 9$
   $30x + 50y = -90$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

GreatHearts®

Wednesday, March 25

Pre-Calculus: chapter 9
Lesson 3: solve system of equations by addition.

Objective: Solve systems of equations by addition method
Interchange any two equations of the system and/or replace any equation in the system by the sum of that equation and a nonzero multiple of any other equation in the system.

Lesson 3:
Given a system of equations, solve using the addition method.

1. Write both equations with x- and y-variables on the left side of the equal sign and constants on the right.
2. Write one equation above the other, lining up corresponding variables. If one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, add the equations together, eliminating one variable. If not, use multiplication by a nonzero number so that one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, then add the equations to eliminate the variable.
3. Solve the resulting equation for the remaining variable.
4. Substitute that value into one of the original equations and solve for the second variable.
5. Check the solution by substituting the values into the other equation

Example: Solve the given system of equations by addition.
Solve the given system of equations by addition.

$$x + 2y = -1$$
$$-x + y = 3$$

Solution Both equations are already set equal to a constant. Notice that the coefficient of x in the second equation, -1, is the opposite of the coefficient of x in the first equation, 1. We can add the two equations to eliminate x without needing to multiply by a constant.

$$\begin{array}{r} x + 2y = -1 \\ -x + y = 3 \\ \hline 3y = 2 \end{array}$$

Now that we have eliminated x, we can solve the resulting equation for y.

$$3y = 2$$
$$y = \frac{2}{3}$$

Then, we substitute this value for y into one of the original equations and solve for x.

$$-x + y = 3$$
$$-x + \frac{2}{3} = 3$$
$$-x = 3 - \frac{2}{3}$$
$$-x = \frac{7}{3}$$
$$x = -\frac{7}{3}$$

The solution to this system is $\left(-\frac{7}{3}, \frac{2}{3}\right)$

Check the solution in the first equation.

$$x + 2y = -1$$
$$\left(-\frac{7}{3}\right) + 2\left(\frac{2}{3}\right) = -1$$
$$-\frac{7}{3} + \frac{4}{3} = -1$$
$$-\frac{3}{3} = -1$$
$$-1 = -1 \quad \text{True}$$

Analysis *We gain an important perspective on systems of equations by looking at the graphical representation. See* ***Figure 5*** *to find that the equations intersect at the solution. We do not need to ask whether there may be a second solution because observing the graph confirms that the system has exactly one solution.*

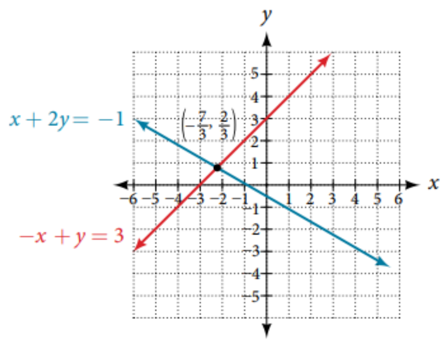


Figure 5

Example: Using the Addition Method When Multiplication of One Equation Is Required

Solve the given system of equations by the addition method.

$$3x + 5y = -11$$
$$x - 2y = 11$$

Solution: Adding these equations as presented will not eliminate a variable. However, we see that the first equation has 3x in it and the second equation has x. So, if we multiply the second equation by −3, the x-terms will add to zero.

$$x - 2y = 11$$
$$-3(x - 2y) = -3(11) \qquad \text{Multiply both sides by } -3.$$
$$-3x + 6y = -33 \qquad \text{Use the distributive property.}$$

Now, let's add them.

$$\begin{aligned} 3x + 5y &= -11 \\ -3x + 6y &= -33 \\ \hline 11y &= -44 \\ y &= -4 \end{aligned}$$

For the last step, we substitute $y = -4$ into one of the original equations and solve for x.

$$\begin{aligned} 3x + 5y &= -11 \\ 3x + 5(-4) &= -11 \\ 3x - 20 &= -11 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

Our solution is the ordered pair (3, −4). See **Figure 6**. Check the solution in the original second equation.

$$\begin{aligned} x - 2y &= 11 \\ (3) - 2(-4) &= 3 + 8 \\ 11 &= 11 \qquad \text{True} \end{aligned}$$

Our solution is the ordered pair (3, −4). See **Figure 6**. Check the solution in the original second equation.

$$\begin{aligned} x - 2y &= 11 \\ (3) - 2(-4) &= 3 + 8 \\ 11 &= 11 \qquad \text{True} \end{aligned}$$

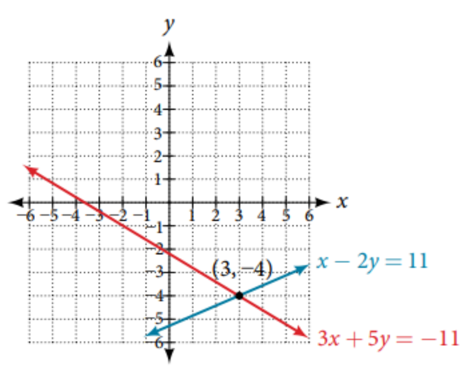


Figure 6

Example: Using the Addition Method When Multiplication of Both Equations Is Required

Solve the given system of equations in two variables by addition.

$$2x + 3y = -16$$
$$5x - 10y = 30$$

Solution: One equation has 2x and the other has 5x. The least common multiple is 10x so we will have to multiply both equations by a constant in order to eliminate one variable. Let's eliminate x by multiplying the first equation by −5 and the second equation by 2.

$$-5(2x+3y)=-5(-16)$$
$$-10x-15y=80$$
$$2(5x-10y)=2(30)$$
$$10x-20y=60$$

Then, we add the two equations together.

$$\begin{aligned} -10x-15y&=80\\ 10x-20y&=60\\ \hline -35y&=140\\ y&=-4 \end{aligned}$$

Substitute $y=-4$ into the original first equation.

$$\begin{aligned} 2x+3(-4)&=-16\\ 2x-12&=-16\\ 2x&=-4\\ x&=-2 \end{aligned}$$

The solution is $(-2,-4)$. Check it in the other equation.

$$\begin{aligned} 5x-10y&=30\\ 5(-2)-10(-4)&=30\\ -10+40&=30\\ 30&=30 \end{aligned}$$

Exercises for March 25, 2020: solve each system by addition.

For the following exercises, solve each system by addition.

1. $-2x+5y=-42$
   $7x+2y=30$

2. $-x+2y=-1$
   $5x-10y=6$

3. $2x+3y=8$
   $3x+5y=10$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Thursday, March 26

Pre-Calculus: chapter 9
Lesson 4: Identifying Inconsistent Systems of Equations Containing Two Variables

Objective: Solve systems of equations in two variables with no solution

Lesson 4

Now that we have several methods for solving systems of equations, we can use the methods to identify inconsistent systems. Recall that an inconsistent system consists of parallel lines that have the same slope but different y-intercepts. They will never intersect. When searching for a solution to an inconsistent system, we will come up with a false statement, such as 12 = 0.

Example: Solving an Inconsistent System of Equations

Solve the following system of equations.

$$\begin{aligned} x &= 9 - 2y \\ x + 2y &= 13 \end{aligned}$$

Solution We can approach this problem in two ways. Because one equation is already solved for x, the most obvious step is to use substitution.

$$\begin{aligned} x + 2y &= 13 \\ (9 - 2y) + 2y &= 13 \\ 9 + 0y &= 13 \\ 9 &= 13 \end{aligned}$$

Clearly, this statement is a contradiction because $9 \neq 13$. Therefore, the system has no solution.

The second approach would be to first manipulate the equations so that they are both in slope-intercept form. We manipulate the first equation as follows.

$$\begin{aligned} x &= 9 - 2y \\ 2y &= -x + 9 \\ y &= -\frac{1}{2}x + \frac{9}{2} \end{aligned}$$

We then convert the second equation expressed to slope-intercept form.

$$\begin{aligned} x + 2y &= 13 \\ 2y &= -x + 13 \\ y &= -\frac{1}{2}x + \frac{13}{2} \end{aligned}$$

Comparing the equations, we see that they have the same slope but different y-intercepts. Therefore, the lines are parallel and do not intersect.

$$\begin{aligned} y &= -\frac{1}{2}x + \frac{9}{2} \\ y &= -\frac{1}{2}x + \frac{13}{2} \end{aligned}$$

Analysis: Writing the equations in slope-intercept form confirms that the system is inconsistent because all lines will intersect eventually unless they are parallel. Parallel lines will never intersect; thus, the two lines have no points in common. The graphs of the equations in this example are shown in Figure 8.

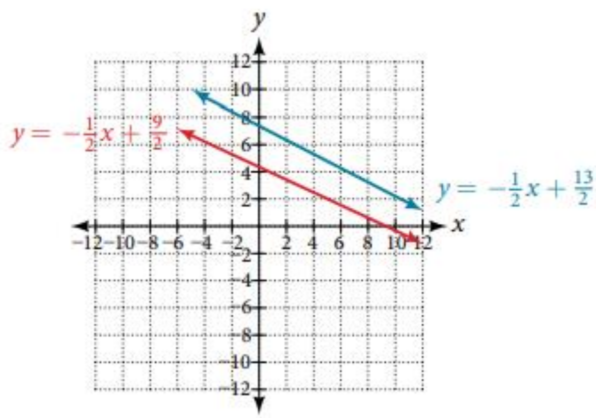


Figure 8

Exercise for Thursday, March 26, 2020

Solve the following system of equations in two variables.

$$2y - 2x = 2$$
$$2y - 2x = 6$$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

GreatHearts®

Friday, March 27

Pre-Calculus: chapter 9
Lesson 5: Expressing the Solution of a System of Dependent Equations Containing Two Variables

Objective: Solve systems of equations in two variables which are lines that are coincident

Lesson 5
Recall that a dependent system of equations in two variables is a system in which the two equations represent the same line. Dependent systems have an infinite number of solutions because all the points on one line are also on the other line. After using substitution or addition, the resulting equation will be an identity, such as 0 = 0.

Example: Finding a Solution to a Dependent System of Linear Equations

Find a solution to the system of equations using the addition method.

$$x + 3y = 2$$
$$3x + 9y = 6$$

Solution: With the addition method, we want to eliminate one of the variables by adding the equations. In this case, let's focus on eliminating x. If we multiply both sides of the first equation by −3, then we will be able to eliminate the x-variable.

$$\begin{aligned} x + 3y &= 2 \\ (-3)(x + 3y) &= (-3)(2) \\ -3x - 9y &= -6 \end{aligned}$$

Now add the equations.

$$\begin{array}{rr} & -3x - 9y = -6 \\ + & 3x + 9y = 6 \\ \hline & 0 = 0 \end{array}$$

We can see that there will be an infinite number of solutions that satisfy both equations.

Analysis: If we rewrote both equations in the slope-intercept form, we might know what the solution would look like before adding. Let's look at what happens when we convert the system to slope-intercept form.

$$\begin{aligned} x + 3y &= 2 \\ 3y &= -x + 2 \\ y &= -\frac{1}{3}x + \frac{2}{3} \\ 3x + 9y &= 6 \\ 9y &= -3x + 6 \\ y &= -\frac{3}{9}x + \frac{6}{9} \\ y &= -\frac{1}{3}x + \frac{2}{3} \end{aligned}$$

*See **Figure 9**. Notice the results are the same. The general solution to the system is* $\left(x, -\frac{1}{3}x + \frac{2}{3}\right)$.

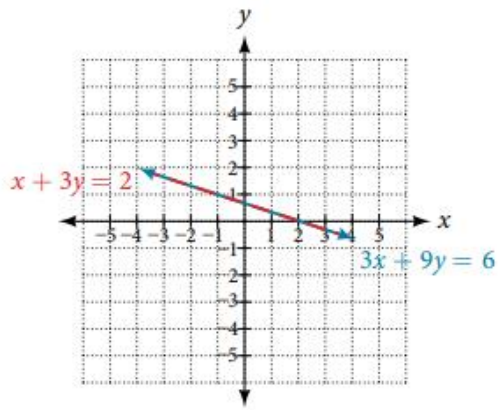


Figure 9

Exercise for March 27, 2020

Solve the following system of equations in two variables.

$$y - 2x = 5$$
$$-3y + 6x = -15$$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

ANSWER KEY

Exercises for Monday, March 23, 2020

1.) What are the coefficients of the equation 2x – 5y = –23 ? 2, -5
2.) What is the constant of the equation 2x – 5y = –23 ? –23
3.) Is x = –4 and y = 3 a solution of the equation 2x –5y = –23? 2(–4) –5(3) = –23 True, yes it is a solution
4.) What are the coefficients of the equation –7x + 6y = 15 ? –7, 6
5.) What is the constant of the equation –7x + 6y = 15? 15
6.) Is x = 3 and y = –10 a solution of the equation –7x + 6y = 15? –7(3) + 6(–10) = 15 FALSE
7.) Is x = 1 and y = 0 a solution of the system: NO

x + y = 1
2x + 3y = 3

8.) Is x = –1 and y = 3 a solution of the system

7x + 2y = –1 → 7(-1) + 2(3) = –1
5x – 3y = –14 → 5(-1) – 3(3) = –14
YES, x = –1 and y = 3 a solution.

Exercises for March 24, 2020: Solve using the substitution method

1. (-1,2)
2. (-3,1)
3. No solutions exist

Exercises for March 25, 2020: solve each system by addition.

1. (6,-6)
2. No solutions exist
3. (10,–4)

Exercises for Thursday, March 26, 2020

No solution. It is an inconsistent system.

Exercise for March 27, 2020

The system is dependent so there are infinite solutions of the form (x, 2x + 5).

NOTE: **At a later point, we will ask you to turn your exercise packet. Do not worry right now about whether that will be online or in person, simply do the problems as I instruct with the proper titles and labels.**