Pre-Algebra 7: Square Roots and Pythagorean Theorem

March 23-27

Time Allotment: 40 minutes per day

Student Name: ________________________________

Teacher Name: __________________________________

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:
### Packet Overview

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### Additional Notes:

Greetings Students (and Parents!). We miss you all very much! Though we cannot enjoy being together, we can nonetheless still share the joy of exploring math together. This will obviously require a lot of independent work and study, but we are here to support and guide you in every way we can. As with class, you need to ask questions! Please e-mail your teacher at either Melisa.Walters@greatheartsnorthernoaks.org or Patrick.Franzes@greatheartsnorthernoaks.org.

Each lesson will end with a set of math problems pertaining to that particular lesson of the day. For the problems that are assigned from the textbook pages that are copied into this packet, you must complete the assigned problems on a blank sheet of paper. You should start each day’s assignment on a separate piece of paper that includes:

1. The title and date of the lesson
2. The notes, if any, the student was directed to copy down
3. The assigned problems with appropriate work show

Then, at the end of the week, you should staple those assignments together—along complete last page of this packet which is the quiz and Wednesday’s assigned problems—to turn in.

The answer key for all assigned problems (except for the Quiz!) will be found at the back of the packet. You should use the answer key to check and grade your answers. Note, in order to receive credit for completing the assigned problems, you must show work where appropriate.

While you may use a calculator on assigned problems, you may not use one on the quiz.

Note, we will ultimately have a test on these concepts, although that date has not been determined. You were already provided a study guide, but one is provided at the end of this packet as well.

Each day’s lesson is designed to take no more than 40 minutes. As a general rule, reading the lesson and taking notes should take approximately 20 minutes and completing the problems should take approximately 20 minutes. If you have spent more than 40 minutes on a lesson, then have your parent sign the paper where you are doing assigned problems on for that lesson and you will still receive full credit for completing them.

Finally, if you are looking for additional resources to help you learn and explore Pre-Algebra during this time, please ask! Many of you are probably already aware of Khan Academy, which is a wonderful resource, and we encourage you to use it.
Monday, March 23
Unit: Square Roots and Right Triangles
Lesson 4: Square Roots, Pythagorean Theorem and Pythagorean Triples

Objective: Be able to do this by the end of this lesson.
1. Simplify Square Roots
2. Solve problems using the Pythagorean Theorem
3. Identify Pythagorean Triples

Introduction to Lesson
Prior to Spring Break, we studied square roots and the Pythagorean Theorem. This lesson will review those concepts and further highlight a special class of right triangles that have three sides which are referred to as Pythagorean Triples.

Square Roots

Recall that we can write \( b \times b \) as \( b^2 \) and call it the square of \( b \). The square root of \( b^2 \) is the factor \( b \). A given number \( a \) has \( b \) as a square root if

\[
    b^2 = a.
\]

Thus 9 has 3 as a square root because \( 3^2 = 9 \).

Every positive number has two square roots, and these are opposites of each other. For example, the square roots of 25 are 5 and \(-5\) because

\[
    5^2 = 5 \times 5 = 25 \quad \text{and} \quad (-5)^2 = (-5) \times (-5) = 25.
\]

The only square root of 0 is 0 because \( b \times b = 0 \) only when \( b = 0 \).

In this chapter we will work mostly with positive square roots. We use \( \sqrt{a} \) to denote the positive square root of \( a \). Thus \( \sqrt{25} = 5 \), not \(-5\). A symbol such as \( 2\sqrt{25} \) means \( 2 \) times the positive square root of 25.

The negative square root of 25 is \(-\sqrt{25} \), or \(-5\).

Negative numbers have no real-number square roots because no real number has a square that is negative.

If \( \sqrt{a} \) is an integer, we call \( a \) a perfect square. For example, 36 is a perfect square because \( \sqrt{36} \) is the integer 6. Also, 144 is a perfect square because \( \sqrt{144} = 12 \).

If \( a \) is not a perfect square, we can estimate \( \sqrt{a} \) by finding the two consecutive integers between which the square root lies. In the process, we use the fact that the smaller of two positive numbers has the smaller positive square root.

**EXAMPLE** Between which two consecutive integers does \( \sqrt{40} \) lie?

**Solution** 40 lies between the consecutive perfect squares 36 and 49.

\[
    36 < 40 < 49
\]

\[
    \sqrt{36} < \sqrt{40} < \sqrt{49}
\]

Thus \( 6 < \sqrt{40} < 7 \).
To highlight, the symbol $2\sqrt{49}$ means $2 \times \sqrt{49}$, which can be simplified to $2 \times 7$ or 14.
Moreover, we can easily multiply square roots. For example, we can simplify $\sqrt{20} \times \sqrt{5}$ by looking at the problem as $\sqrt{20} \times \sqrt{5}$ which can simplify as $\sqrt{100}$ or 10. To put both of these ideas together, if you have a problem such as “Simplify $2\sqrt{5} \times \sqrt{20}$” we can use the associative property of multiplication as follows: $2\sqrt{5} \times \sqrt{20}$ is the same as $(2)(\sqrt{5})(\sqrt{20})$. This can further be simplified to $(2)(\sqrt{5 \times 20}) = (2)(\sqrt{100}) = (2)(10) = 20$.

**First Assignment for Monday, March 23:**
You must complete the assigned problems on another sheet of paper to turn in.
As discussed before break, you should work on memorizing perfect squares 1-20 (i.e., $1^2 = 1$; $2^2 = 4$; $3^2 = 9$; $4^2 = 16$... up to 20). Fill in as many answers as you can without using a calculator. Then check your answers and spend five minutes memorizing the ones you got wrong.

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**Second Assignment for Monday, March 23:**
You must complete the assigned problems on another sheet of paper to turn in.
WE, #2-34 Even

**Written Exercises**
If the given symbol names an integer, state the integer. If not, name the two consecutive integers between which the number lies.

<table>
<thead>
<tr>
<th>A</th>
<th>1. $\sqrt{45}$</th>
<th>2. $\sqrt{64}$</th>
<th>3. $-\sqrt{16}$</th>
<th>4. $\sqrt{24}$</th>
<th>5. $\sqrt{1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>$\sqrt{6}$</td>
<td>7. $-\sqrt{6}$</td>
<td>8. $\sqrt{13}$</td>
<td>9. $\sqrt{54}$</td>
<td>10. $\sqrt{9}$</td>
</tr>
<tr>
<td>11.</td>
<td>$\sqrt{50}$</td>
<td>12. $\sqrt{48}$</td>
<td>13. $\sqrt{15}$</td>
<td>14. $\sqrt{8}$</td>
<td>15. $\sqrt{2}$</td>
</tr>
<tr>
<td>16.</td>
<td>$\sqrt{20} + \sqrt{16}$</td>
<td>17. $\sqrt{100} - \sqrt{49}$</td>
<td>18. $\sqrt{144} + \sqrt{25}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.</td>
<td>$\sqrt{9} - 6$</td>
<td>20. $-\sqrt{66} - \sqrt{2}$</td>
<td>21. $\sqrt{100} - 19$</td>
<td></td>
<td></td>
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</tbody>
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Replace the _?_ with _, >, or = to make a true statement.

**EXAMPLE**
$\sqrt{9} + \sqrt{25}$ _?_ $\sqrt{9} + 25$

**Solution**
$\sqrt{9} + \sqrt{25} = 3 + 5 = 8$; $\sqrt{9} + 25 = \sqrt{34} < 8$.
Thus $\sqrt{9} + \sqrt{25} > \sqrt{9} + 25$.

**B**
22. $\sqrt{9} + \sqrt{16}$ _?_ $\sqrt{9} + 16$
23. $\sqrt{16} + \sqrt{4}$ _?_ $\sqrt{16} + 4$
24. $\sqrt{16} - \sqrt{9}$ _?_ $\sqrt{16} - 9$
25. $\sqrt{25} - \sqrt{9}$ _?_ $\sqrt{25} - 9$
26. $\sqrt{4} \times \sqrt{9}$ _?_ $\sqrt{4} \times 9$
27. $\sqrt{25} \times \sqrt{9}$ _?_ $\sqrt{25} \times 4$
28. $2\sqrt{2}$ _?_ $\sqrt{2} \times 2$
29. $3\sqrt{12}$ _?_ $\sqrt{3} \times 12$

Evaluate the expression.

C
30. $(\sqrt{25})^2$ 31. $(\sqrt{81})^2$ 32. $(\sqrt{49})^2$ 33. $(\sqrt{11})^2$ 34. $(\sqrt{5})^2$

You can check your answers on page 15 to see how you did!
The longest side of a right triangle is opposite the right angle and is called the hypotenuse. The two shorter sides are called legs.

About 2500 years ago, the Greek mathematician Pythagoras proved the following useful fact about right triangles.

**The Pythagorean Theorem**

If the hypotenuse of a right triangle has length $c$, and the legs have lengths $a$ and $b$, then

$$c^2 = a^2 + b^2.$$ 

The figure at the right illustrates the Pythagorean theorem. We see that the area of the square on the hypotenuse equals the sum of the areas of the squares on the legs:

$$25 = 9 + 16,$$

or

$$5^2 = 3^2 + 4^2.$$ 

The converse of the Pythagorean theorem is also true. It can be used to test whether a triangle is a right triangle.

**Converse of the Pythagorean Theorem**

If the sides of a triangle have lengths $a$, $b$, and $c$, such that $c^2 = a^2 + b^2$, then the triangle is a right triangle.

**Example 1**

Is the triangle with sides of the given lengths a right triangle?

- a. 4, 5, 7
- b. 5, 12, 13

**Solution**

- a. $4^2 = 16$, $5^2 = 25$, $7^2 = 49$

  No, since $16 + 25 \neq 49$. The triangle with sides of lengths 4, 5, and 7 **cannot** be a right triangle.
Pythagorean Triples
A Pythagorean Triple consists of three positive integers $a$, $b$, and $c$ (i.e., whole numbers excluding zero) that satisfies the equation $a^2 + b^2 = c^2$. The most common Pythagorean Triple is 3, 4, and 5 since $3^2 + 4^2 = 5^2$. In fact, many of the examples we looked at in class were Pythagorean Triples because we wanted to use problems that ensured each side of the right triangle was a positive integer.

Third Assignment for Monday, March 23:
You must complete the assigned problems on another sheet of paper to turn in.
WE, #4-16 Even

Any questions? E-mail your teacher at either Melisa.Walters@greatheartsnorthernoaks.org or Patrick.Franzese@greatheartsnorthernoaks.org.
Tuesday, March 24
Unit: Square Roots and Right Triangles
Lesson 5: Similar Triangles

Objective: Be able to do this by the end of this lesson.
1. Identify similar triangles
2. Solve problem by applying the properties of similar triangles

Introduction to Lesson
Earlier this year when we studied congruent polygons, congruence was defined as two figures having the same size and shape. In this lesson, we consider what can still be said to be true if two figures have the same shape but not necessarily the same size. In doing so, we will be using ratios and proportions, concepts that we have studied earlier as well. This lesson seems out of place when considering how this unit began with square roots and the Pythagorean theorem. However, in the next lesson, the properties of similar figures will be united with the Pythagorean theorem to discuss special right triangles. With that perspective, this lesson can be seen as the second of two necessary threads leading to the next lesson.

Similar Triangles

As you know, we say that two figures are congruent when they are identical in both shape and size. When two figures have the same shape, but do not necessarily have the same size, we say that the figures are similar.

For two triangles to be similar it is enough that the measures of their corresponding angles are equal.

\[
\triangle ABC \sim \triangle DEF
\]

The symbol \(\sim\) means is similar to. Note that when we write expressions such as the one above, we list corresponding vertices in the same order.

In triangles \(ABC\) and \(DEF\), we see that the lengths of the corresponding sides have the same ratio:

\[
\frac{AB}{DE} = \frac{18}{12} = \frac{3}{2} \quad \frac{BC}{EF} = \frac{12}{8} = \frac{3}{2} \quad \frac{CA}{FD} = \frac{13}{10} = \frac{3}{2}
\]

Therefore,

\[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}
\]

Because the ratios are equal, we say that the lengths of the corresponding sides are proportional.

In general, we can say the following.

For two similar triangles, corresponding angles are congruent and lengths of corresponding sides are proportional.
Since this is a new concept, to help re-enforce it, you must copy down into your daily assignment sheet that is going to be turned in the items that are circled below.

**Example 1**

If $\triangle RUN \sim \triangle JOG$, find the lengths marked $x$ and $y$.

**Solution**

Since the corresponding vertices are listed in the same order, we know the following:

- $\angle R \cong \angle J$
- $\angle U \cong \angle O$
- $\angle N \cong \angle G$

\[
\frac{RU}{JO} = \frac{UN}{OG} = \frac{NR}{GJ}
\]

Substituting the values that are given in the diagram, we obtain

\[
\frac{x}{40} = \frac{15}{24} = \frac{35}{y}
\]

Therefore, we can set up one proportion involving $x$ and another involving $y$.

\[
\begin{align*}
\frac{x}{40} &= \frac{15}{24} & \frac{15}{24} &= \frac{35}{y} \\
24x &= 15 \times 40 & 15y &= 35 \times 24 \\
x &= \frac{15 \times 40}{24} & y &= \frac{35 \times 24}{15} \\
x &= \frac{600}{24} = 25 & y &= \frac{840}{15} = 56
\end{align*}
\]

The length of $x$ is 25 and the length of $y$ is 56.

In the two right triangles shown at the right, the measures of two acute angles are equal. Since all right angles have equal measure, $90^\circ$, the two remaining acute angles must also have equal measure. (Recall that the sum of the angle measures of any triangle is $180^\circ$.) Thus the two right triangles are similar.

If an acute angle of one right triangle is congruent to an angle of a second right triangle, then the triangles are similar.
EXAMPLE 2. At the same time a tree on level ground casts a shadow 48 m long, a 2 m pole casts a shadow 5 m long. Find the height, $h$, of the tree.

Solution. In the diagram, right triangles $ABC$ and $ADE$ share acute $\angle A$ and thus are similar. Since $\frac{AD}{AB} = \frac{DE}{BC} = \frac{EA}{CA}$, we can set up a proportion to solve for $h$.

$$\frac{h}{2} = \frac{48}{5} \quad \Rightarrow \quad 5h = 48 \times 2 \quad \Rightarrow \quad h = \frac{48 \times 2}{5} = 19.2$$

The height of the tree is 19.2 m.

COMMUNICATION IN MATHEMATICS: Reading Diagrams

To help you read a diagram that has overlapping triangles, you can redraw the diagram, pulling apart the individual triangles. For example, $\triangle ADE$ above can be separated as shown below.

Class Exercises

In Exercises 1–4, $\triangle LOG \sim \triangle RIT$.

1. Name all pairs of corresponding angles.
2. Name all pairs of corresponding sides.
3. $\frac{OL}{IR} = \frac{OG}{IT}$
4. $\frac{LG}{IT} = \frac{GO}{IT}$

True or false?
5. $\triangle BAE$ and $\triangle DCE$ are right triangles.
6. $m \angle ABE = m \angle CDE$
7. $\triangle BAE \sim \triangle DCE$
8. $\frac{CE}{AE} = \frac{BA}{DE}$
9. $\frac{BA}{DC} = \frac{BE}{DE}$

You do not need to do the CE in your notes! But below are the answers to the above CE.
Assignment for Tuesday, March 24:
You must complete the assigned problems on another sheet of paper to turn in.
WE, #1-20 ALL

Written Exercises

Exercises 1–5 refer to the diagram at the right. \( \triangle ABC \sim \triangle PQR. \)

1. \( \frac{PR}{AC} = \frac{?}{CB} \)

2. \( \frac{BA}{?} = \frac{CB}{RQ} \)

3. Find the length of \( RQ \).

4. Find the length of \( AC \).

5. \( m \angle A = 47^\circ \) and \( m \angle Q = 29^\circ \),
   \( m \angle C = \ ?^\circ \).

Exercises 6–10 refer to the diagram at the right. \( \triangle AED \sim \triangle ACB. \)

6. The length of \( AE = \ ? \).

7. The length of \( AB = \ ? \).

8. The length of \( AD = \ ? \).

9. If \( m \angle A = 25^\circ \), then \( m \angle ABC = \ ?^\circ \)
   and \( m \angle ADE = \ ?^\circ \).

10. If \( \frac{BC}{DE} = \frac{1}{2} \), then \( \frac{AB}{AD} = \ ? \).

Find the lengths marked \( x \) and \( y \).

11. \( \triangle MAB \sim \triangle SRO \)

12. \( \triangle DIP \sim \triangle MON \)

13. \( \triangle RST \sim \triangle RUV \)

14. \( \triangle GEM \sim \triangle TIM \)

15. \( \triangle PAR \sim \triangle PBT \)

16. \( \triangle ART \sim \triangle ABC \)
Any questions? E-mail your teacher at either Melisa.Walters@greatheartsnorthernoaks.org or Patrick.Franzese@greatheartsnorthernoaks.org.

**Wednesday, March 24**

Unit: Square Roots and Right Triangles
Lesson 6: Quiz and Similar Triangles Problems

**Objective:** Be able to do this by the end of this lesson.
1. Demonstrate proficiency in using square roots and the Pythagorean Theorem.

**Introduction to Lesson**

Today’s lesson will consist:
1. A quiz. *Closed notes and no calculator!*
2. Word problems that build upon the yesterday’s lesson on similar triangles.

The quiz is on page 23 of this packet. After completing the quiz, complete the similar triangles word problems that are on page 24 of this packet. You should tear this last page out (pgs 23-24) and staple it to the other completed assigned problems that need to be turned in.

Any questions? E-mail your teacher at either Melisa.Walters@greatheartsnorthernoaks.org or Patrick.Franzese@greatheartsnorthernoaks.org.
Thursday, March 24
Unit: Square Roots and Right Triangles
Lesson 7: Special Right Triangles

Objective: Be able to do this by the end of this lesson.
1. Calculate the missing side lengths of an isosceles right triangle when given one of the sides.
2. Calculate the missing length of a 30-60-90 right triangle when given one of the side lengths.

Introduction to Lesson
In the previous lessons we worked with the Pythagorean Theorem and with similar triangles. In this lesson, we combine topics to discuss special right triangles. This lesson is an excellent opportunity for problem solving and discovering the ratios of the sides of the 45-45-90 and the 30-60-90 right triangles. Note, this lesson will be challenging for students and is designed to stretch them. The concepts from this lesson will not be on the test.

Special Right Triangles

***Since this is a new concept, to help re-enforce it, you must copy down into your daily assignment sheet that is going to be turned in the items that are circled below***

---

10-6 Special Right Triangles

In an isosceles right triangle the two acute angles are congruent. Since the sum of the measures of these two angles is 90°, each angle measures 45°. For this reason, an isosceles right triangle is often called a 45° right triangle.

In the diagram each leg is 1 unit long. If the hypotenuse is $c$ units long, by the Pythagorean theorem we know that $c^2 = 1^2 + 1^2 = 2$ and thus

$$c = \sqrt{2}.$$

Every 45° right triangle is similar to the one shown. Since corresponding sides of similar triangles are proportional, we have the following property.

If each leg of a 45° right triangle is $a$ units long, then the hypotenuse is $a\sqrt{2}$ units long.
EXAMPLE 1  A square park measures 200 m on each edge. Find the length, \(d\), of a path extending diagonally from one corner to the opposite corner. Use \(\sqrt{2} \approx 1.414\).

Solution  
Since the park is square, we can apply the property of 45° right triangles to solve for \(d\). We use the fact that \(d\) is the hypotenuse of the right triangle and that each side measures 200 m.

\[
d = 200 \sqrt{2} \approx 200 \times 1.414 \approx 282.8
\]

Thus the path measures approximately 282.8 m.

A 30°-60° right triangle, such as \(\triangle ABC\), may be thought of as half an equilateral triangle. If hypotenuse \(AB\) is 2 units long, then the shorter leg \(AC\) (half of \(AD\)) is 1 unit long. To find \(BC\), we use the Pythagorean theorem:

\[
(AC)^2 + (BC)^2 = (AB)^2
\]

\[
1^2 + (BC)^2 = 2^2
\]

\[
(BC)^2 = 4 - 1 = 3
\]

\[
BC = \sqrt{3}
\]

***Since this is a new concept, to help re-enforce it, you must copy down into your daily assignment sheet that is going to be turned in the items that are circled below***

Every 30°-60° right triangle is similar to the one above, and since corresponding sides of similar triangles are proportional, we have the following property.

If the shorter leg of a 30°-60° right triangle is \(a\) units long, then the longer leg is \(a\sqrt{3}\) units long, and the hypotenuse is \(2a\) units long.

EXAMPLE 2  The hypotenuse of a 30°-60° right triangle is 8 cm long. Find the lengths of the legs.

Solution  
Using the 30°-60° right triangle property, we know that

\[
2a = 8, \quad a = 4, \quad \text{and} \quad a\sqrt{3} = 4\sqrt{3}.
\]

Thus the lengths of the legs are 4 cm and \(4\sqrt{3}\) cm.

The symbol \(\sqrt{\cdot}\) is called the radical sign. An expression such as \(\sqrt{3}\) or \(\sqrt{x}\) is called a radical. We often leave answers in terms of radicals with the radical in the numerator. To rewrite expressions such as \(\frac{6}{\sqrt{3}}\) so that the radical appears in the numerator, we may use the fact that \(\sqrt{x} \times \sqrt{x} = x\).
Assignment for Thursday, March 26:
You must complete the assigned problems on another sheet of paper to turn in.
WE, #6-12 & 18-22

Follow these steps to solve the problems #7-12:

1. Identify the location of the 90 degree angle
2. Directly across from the 90 degree angle is the hypotenuse. According to the picture above the hypotenuse is equal to 2x. So, for problem #7, we set 2x = 6.4, Then, x = 3.2
3. In problem 7, the x is adjacent to the 60 degree angle, so we will label x = 3.2
4. In problem #7, y = x\sqrt{3} = 3.2\sqrt{3} = 3.2(1.732) = 5.5

Therefore, x = 3.2, y = 5.5.

Approximate the lengths marked x and y to the nearest tenth.
Use \sqrt{2} \approx 1.414 and \sqrt{3} \approx 1.732.
Follow these steps for problems #17 – 22:

1. Draw a right triangle ABC labeled accordingly:

2. We are given that side AC = 2 and side BC = 5

3. We apply the Pythagorean theorem: \( a^2 + b^2 = c^2 \) to find the last side. The last side to calculate is the side AB known as the hypotenuse in problem #17.

\[
\begin{align*}
   a^2 + b^2 &= c^2 \\
   2^2 + 5^2 &= c^2 \\
   4 + 25 &= c^2 \\
   29 &= c^2 \\
   c &= 5.4
\end{align*}
\]

In Exercises 17–22, \( \angle C \) is a right angle in \( \triangle ABC \). Find the length of the missing side to the nearest tenth.

17. \( AC = 2, BC = 5 \)
18. \( AB = 18, AC = 9 \)
19. \( AB = 6\sqrt{2}, AC = 6 \)
20. \( AC = CB = x \)
21. \( \frac{1}{2} BA = CA = y \)
22. \( 2BC = BA = z \)

Any questions? E-mail your teacher at either Melisa.Walters@greatheartsnorthernoaks.org or Patrick.Franzese@greatheartsnorthernoaks.org.

**Friday, March 27**
Unit: Square Roots and Right Triangles
Lesson 8: Review

**Objective:** Be able to do this by the end of this lesson.
1. Demonstrate proficiency in working with square roots and using the Pythagorean Theorem

**Introduction to Lesson**
This lesson is a review of the concepts we have studied during this unit. Completing the questions below will ensure you have gained a solid understanding of the concepts in this unit and will prepare you for next week’s test.
Assignment for Friday, March 27:
You must complete the assigned problems on another sheet of paper to turn in.
Chapter Review, #1-2, 7-15

Chapter Review

Complete.
1. A positive number has exactly 2 different square root(s).
2. \( \sqrt{169} \) is 13, therefore 169 is a square.
3. If 5.2 is used as an estimate for \( \sqrt{28.6} \) in the divide-and-average method, the next estimate will be 2.2.
4. Using the divide-and-average method, \( \sqrt{53} = 7.3 \) to the nearest tenth.
5. In the table on page 528, \( \sqrt{24.6} \) lies between 4.7 and 4.8.
6. Using interpolation and the table on page 528, \( \sqrt{5.7} = 2.4 \) to the nearest hundredth.

True or false?
7. The Pythagorean theorem applies to all triangles.
8. The hypotenuse is the longest side of a right triangle and is opposite the right angle.
9. The measure of the diagonal of a 5 cm by 5 cm square is 50 cm.

Exercises 10-12 refer to the diagram below. \( \triangle ABC \sim \triangle DEF \). Complete.

10. \( \frac{BA}{?} = \frac{AC}{DF} \)
11. \( \angle C \approx \angle ? \)
12. \( BC \) corresponds to ?.
13. An isosceles right triangle is also called a \( 45^\circ \) right triangle.
14. An equilateral triangle with sides 16 cm long has an altitude of \( ? \) cm.
15. The legs of an isosceles right triangle are 7 cm long. The length of the hypotenuse is \( ? \) cm.
Chapter Test

If the given symbol names an integer, state the integer. If not, name the two consecutive integers between which the number lies.

1. $\sqrt{16}$  
2. $-\sqrt{144}$  
3. $\sqrt{169} - \sqrt{121}$  
4. $\sqrt{38} + 43$

Solve. Give your answer to the nearest tenth.

5. Use the divide-and-average method to approximate $\sqrt{13.7}$.
6. Using interpolation and the table on page 528, $\sqrt{12.6} = \ ?$.
7. The area of a square deck is 65.6 m². Find the length of the side.

Is the triangle with sides of the given lengths a right triangle?

8. 6, 8, 10  
9. 7, 11, 19  
10. 8, 15, 17

Exercises 11–13 refer to the diagram at the right. $	riangle MNO \sim \triangle XYZ$.

11. If $\frac{MN}{XY} = \frac{3}{4}$, then $\frac{NO}{YZ} = \ ?$.
12. Find the length of $\overline{MO}$.
13. Find the measure of $\angle N$.

Give answers in terms of radicals with the radical in the numerator.

14. In a $30^\circ$–$60^\circ$ right triangle, the shorter leg has length 5. How long is (a) the longer leg and (b) the hypotenuse?
15. The hypotenuse of a $45^\circ$ right triangle has a length of 36. How long is each leg?

Any questions? E-mail your teacher at either Melisa.Walters@greatheartsnorthernoaks.org or Patrick.Franzese@greatheartsnorthernoaks.org.
Assignment KEY for Monday, March 23:
(1) WE, #2-34 Even

Page 357 • WRITTEN EXERCISES

A 1. $36 < 43 < 49, \sqrt{36} < \sqrt{43} < \sqrt{49}, 6 < \sqrt{43} < 7$
   2. $\sqrt{64} = 8$
   3. $-\sqrt{16} = -4$
   4. $16 < 24 < 25, \sqrt{16} < \sqrt{24} < \sqrt{25}, 4 < \sqrt{24} < 5$
   5. $\sqrt{1} = 1$
   6. $\sqrt{0} = 0$
   7. $-\sqrt{6^2} = -\sqrt{36} = -6$
   8. $9 < 13 < 16, \sqrt{9} < \sqrt{13} < \sqrt{16}, 3 < \sqrt{13} < 4$
   9. $49 < 54 < 64, \sqrt{49} < \sqrt{54} < \sqrt{64}, 7 < \sqrt{54} < 8$
  10. $\sqrt{9} = 3$
  11. $25 < 30 < 36, \sqrt{25} < \sqrt{30} < \sqrt{36}, 5 < \sqrt{30} < 6$
  12. $36 < 48 < 49, \sqrt{36} < \sqrt{48} < \sqrt{49}, 6 < \sqrt{48} < 7$
  13. $9 < 15 < 16, \sqrt{9} < \sqrt{15} < \sqrt{16}, 3 < \sqrt{15} < 4$
  14. $\sqrt{8^2} = \sqrt{64} = 8$
  15. $1 < 2 < 4, \sqrt{1} < \sqrt{2} < \sqrt{4}, 1 < \sqrt{2} < 2$
  16. $\sqrt{25} + \sqrt{16} = 5 + 4 = 9$
  17. $\sqrt{100} - \sqrt{49} = 10 - 7 = 3$
  18. $\sqrt{144} + \sqrt{25} = 12 + 5 = 17$
  19. $\sqrt{79} - 61 - \sqrt{18}; 16 < 18 < 25, \sqrt{16} < \sqrt{18} < \sqrt{25}, 4 < \sqrt{79} - 61 < 5$
  20. $-\sqrt{66} - 2 = -\sqrt{64} = -8$
  21. $\sqrt{100} - 19 = \sqrt{81} = 9$

B 22. $\sqrt{9} + \sqrt{16} = 3 + 4 = 7; \sqrt{9} + \sqrt{16} = \sqrt{25} = 5 < 7$. Thus $\sqrt{9} + \sqrt{16} > \sqrt{9} + \sqrt{16}$.
  23. $\sqrt{16} + \sqrt{4} = 4 + 2 = 6; \sqrt{16} + 4 = \sqrt{20} < 6$. Thus $\sqrt{16} + \sqrt{4} > \sqrt{16} + 4$.
  24. $\sqrt{16} - \sqrt{9} = 4 - 3 = 1; \sqrt{16} - \sqrt{9} = \sqrt{7} > 1$. Thus $\sqrt{16} - \sqrt{9} < \sqrt{16} - \sqrt{9}$.
  25. $\sqrt{25} - \sqrt{9} = 5 - 3 = 2; \sqrt{25} - 9 = \sqrt{16} = 4 > 2$. Thus $\sqrt{25} - \sqrt{9} < \sqrt{25} - 9$.
  26. $\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6; \sqrt{4} \times \sqrt{9} = \sqrt{36} = 6$. Thus $\sqrt{4} \times \sqrt{9} = \sqrt{4} \times \sqrt{9}$.
  27. $\sqrt{25} \times \sqrt{4} = 5 \times 2 = 10; \sqrt{25} \times \sqrt{4} = \sqrt{100} - 10$. Thus $\sqrt{25} \times \sqrt{4} = \sqrt{25} \times \sqrt{4}$.

C 30. $(\sqrt{25})^2 = (5)^2 = 25$
  31. $(\sqrt{81})^2 = (9)^2 = 81$
  32. $(\sqrt{49})^2 = (7)^2 = 49$
  33. $(\sqrt{11})^2 = (\sqrt{11})(\sqrt{11}) = \sqrt{11} \times 11 = \sqrt{121} = 11$
  34. $(\sqrt{2})^2 = (\sqrt{2})(\sqrt{2}) = \sqrt{2} \times 2 = \sqrt{4} = 2
(2) WE, #4-16 Even

Pages 366-367 - WRITTEN EXERCISES

A 1. $A = 8^2 + 15^2 = 64 + 225 = 289$
   2. $A = 12^2 + 5^2 = 144 + 25 = 169$
   3. $A = 12^2 + 16^2 = 144 + 256 = 400$
   4. $6^2 = 36, 8^2 = 64, 10^2 = 100$. Yes, it is a right triangle, since $36 + 64 = 100$.
   5. $8^2 = 64, 16^2 = 225, 17^2 = 289$. Yes, it is a right triangle, since $64 + 225 = 289$.
   6. $16^2 = 256, 30^2 = 900, 34^2 = 1156$. Yes, it is a right triangle, since $256 + 900 = 1156$.
   7. $9^2 = 81, 12^2 = 144, 15^2 = 225$. Yes, it is a right triangle, since $81 + 144 = 225$.
   8. $1.5^2 = 2.25, 2.0^2 = 4, 2.5^2 = 6.25$. Yes, it is a right triangle, since $2.25 + 4 = 6.25$.
   9. $0.6^2 = 0.36, 0.8^2 = 0.64, 1.0^2 = 1$. Yes, it is a right triangle, since $0.36 + 0.64 = 1$.
   10. $9^2 = 81, 21^2 = 441, 23^2 = 529$. No, it is not a right triangle, since $81 + 441 \neq 529$.
   11. $20^2 = 400, 21^2 = 441, 29^2 = 841$. Yes, it is a right triangle, since $400 + 441 = 841$.
   12. $9^2 = 81, 40^2 = 1600, 41^2 = 1681$. Yes, it is a right triangle, since $81 + 1600 = 1681$.
   13. $8^2 = 64, 37^2 = 1369, 39^2 = 1521$. No, it is not a right triangle, since $64 + 1369 \neq 1521$.

B 14. $c^2 = 2^2 + 1^2 = 4 + 1 = 5; c = \sqrt{5} \approx 2.24$
   15. $c^2 = 8^2 + 6^2 = 64 + 36 = 100; c = \sqrt{100} = 10$
   16. $9^2 = 4^2 + b^2, b^2 = 9^2 - 4^2 = 81 - 16 = 65; b = \sqrt{65} \approx 8.06$
Assignment KEY for Tuesday, March 24:

WE, #1-20 ALL

Pages 371-372 - WRITTEN EXERCISES

A 1. \[ \frac{PR}{AC} = \frac{RQ}{CB} \]

2. \[ \frac{BA}{QP} = \frac{CB}{RQ} \]

3. Let \( RQ = x \), \( \frac{x}{15} = \frac{12}{20} \), \( 20x = 15(12), x = \frac{15(12)}{20}, RQ = 9 \)

4. Let \( AC = x \), \( \frac{x}{6} = \frac{20}{12}, 12x = 6(20), x = \frac{6(20)}{12}, AC = 10 \)

5. \( m \angle A + m \angle B + m \angle C = 180^\circ, m \angle A = 47^\circ, m \angle B = m \angle Q = 29^\circ, 47^\circ + 29^\circ + m \angle C = 180^\circ, m \angle C = 180^\circ - 47^\circ - 29^\circ = 104^\circ \)

6. Let \( AE = x \), \( \frac{x}{24} = \frac{14}{7}, 7x = 14(24), x = \frac{14(24)}{7} = 48; AE = 48 \)

7. Let \( AB = x \), \( x^2 + 24^2 = 49 + 576 = 625, x = \sqrt{625} = 25; AB = 25 \)

8. Let \( AD = x \), \( \frac{x}{25} = \frac{14}{7}, 7x = 14(25), x = \frac{14(25)}{7} = 50; AD = 50 \)

9. \( m \angle A + m \angle ABC + m \angle BCA = 180^\circ, 25^\circ + m \angle ABC + 90^\circ = 180^\circ, m \angle ABC = 180^\circ - 90^\circ - 25^\circ = 65^\circ; m \angle ADE = m \angle ABC = 65^\circ \)

10. If \( \frac{BC}{DE} = \frac{1}{2} \), then \( \frac{AB}{AD} = \frac{1}{2} \)

11. \( \frac{x}{15} = \frac{4}{12}, 12x = 15(4), x = \frac{15(4)}{12}, y = \frac{12}{4}, 4y = 3(12), y = \frac{3(12)}{4} = 9 \)

12. \( \frac{x}{15} = \frac{24}{18}, 18x = 15(24), x = \frac{15(24)}{18}, y = \frac{18}{4}, 24y = 4(18), y = \frac{4(18)}{24} = 3 \)

13. \( \frac{x}{15} = \frac{36}{20}, 20x = 15(36), x = \frac{15(36)}{20}, y = \frac{20}{4}, 36y = 45(20), y = \frac{45(20)}{36} = 25 \)

B 14. \( \frac{x}{10} = \frac{13}{25}, 25x = 10(13), x = \frac{10(13)}{26}, y = \frac{12}{3}, 13y = 12(26), y = \frac{12(26)}{13} = 24 \)

15. \( \frac{x}{36} = \frac{27}{21}, 21x = 36(27), x = \frac{36(27)}{21}, y = \frac{21}{27}, 27y = 18(21), y = \frac{18(21)}{27} = 27 \)

16. \( \frac{x}{10} = \frac{45}{25}, 25x = 10(45), x = \frac{10(45)}{25}, y = \frac{25}{20}, 25y = 20(45), y = \frac{20(45)}{25} = 36 \)

17. Let \( x = \) the father's height. \( \frac{150}{200} = \frac{x}{240}, 200x = 150(240), x = \frac{150(240)}{200} = 180; \) her father is 180 cm tall.

18. Let \( x = \) the height of the light. \( \frac{18}{x} = \frac{3}{3 + 7}, 3x = 1.8(10), x = \frac{1.8(10)}{3} = 6; \) the light is 6 m high.

19. a. \( c^2 = a^2 + b^2 = 9 + 16 = 25; c = \sqrt{25} = 5 \). Let \( y = \) the length of the other base of the trapezoid. \( \frac{3}{4} = \frac{4}{y + 4}, 4y = 3(8), y = \frac{3(8)}{4} = 6 \). Let \( z = \) the length of the fourth side of the trapezoid. \( \frac{5}{5 + z} = \frac{3}{6}, 5(6) = 3(5 + z), 30 = 15 + 3z, 15 = 3z, z = 5 \). The perimeter of the trapezoid equals \( 3 + 4 + 6 + 5 \), or 18.

b. \( A = \frac{1}{2}(3 + 6)4 = \frac{1}{2}(9)(4) = 18 \)

20. Let \( x = AB. \) \( \frac{x}{21} = \frac{25}{15}, 15x = 21(25), x = \frac{21(25)}{15} = 35; AB is 35 m long.
Pre-Algebra 7: Square Roots and Pythagorean Theorem
March 23-27

Assignment KEY for Wednesday, March 25:

1. A tree 24 feet tall casts a shadow 12 feet long. Brad is 6 feet tall. How long is Brad's shadow? (draw a diagram and solve)

2. Triangles EFG and QRS are similar. The length of the sides of EFG are 144, 128, and 112. The length of the smallest side of QRS is 280, what is the length of the longest side of QRS?

3. A 40-foot flagpole casts a 25-foot shadow. Find the shadow cast by a nearby building 200 feet tall.

4. A girl 160 cm tall, stands 360 cm from a lamp post at night. Her shadow from the light is 90 cm long. How high is the lamp post?

5. A tower casts a shadow 7 m long. A vertical stick casts a shadow 0.8 m long. If the stick is 1.2 m high, how high is the tower?

Assignment KEY for Thursday, March 26:

WE, #7-12 & 17-22

7. \(x = \frac{6.4}{2} = 3.2; y = 3.2 \sqrt{3} \approx 3.2(1.732) = 5.5424 \approx 5.5\)

8. \(x = 2.7; y = 2.7 \sqrt{2} \approx 2.7(1.414) = 3.8178 \approx 3.8\)

9. \(x = 4.3 \sqrt{3} \approx 4.3(1.732) = 7.4476 \approx 7.4; y = 2(4.3) = 8.6\)

10. \(x = 2(5.9) = 11.8; y = 5.9 \sqrt{3} \approx 5.9(1.732) = 10.2188 \approx 10.2\)

11. \(x = 3.2; y = 3.2 \sqrt{2} \approx 3.2(1.414) = 4.5248 \approx 4.5\)

12. \(x = \frac{8.8}{2} = 4.4; y = 4.4 \sqrt{3} \approx 4.4(1.732) = 7.6208 \approx 7.6\)
17. \((AB)^2 = 2^2 + 5^2 - 4 + 25 = 29; AB = \sqrt{29} \approx 5.385 \approx 5.4\)

18. \(\triangle ABC\) is a 30° – 60° right triangle. \(AB\), the hypotenuse, is 18 units long. \(AC\), the shorter leg, is 9 units long. \(BC\) must be \(9 \sqrt{3} \approx 9(1.732) = 15.588 \approx 15.6\) units long. 
\(BC = 15.6\)

19. \(\triangle ABC\) is a 45° right triangle. \(AB\), the hypotenuse, is \(6 \sqrt{2}\) units long. The legs, \(AC\) and \(BC\), are each 6 units long. \(BC = 6\)

20. \(\triangle ABC\) is a 45° right triangle. Since the legs, \(AC\) and \(CB\), are \(x\) units long, \(AB\), the hypotenuse, is \(x \sqrt{2} \approx 1.414x \approx 1.4x\) units long. \(AB = 1.4x\)

21. \(\triangle ABC\) is a 30° – 60° triangle. \(AC\), the shorter leg is \(y\) units long. \(BC\) is the longer leg. 
\(BC = y \sqrt{3} \approx y(1.732) \approx 1.7y\)

22. \(\triangle ABC\) is a 30° – 60° triangle. \(AB\), the hypotenuse, is \(z\) units long. \(BC\), the shorter leg, is \(\frac{1}{2}z\) units long. \(AC\), the longer leg, is \(\frac{1}{2}z(\sqrt{3})\) units long; \(AC = \frac{1}{2}z(\sqrt{3}) \approx 0.5z(1.732) = 0.8660z \approx 0.9z\)

Assignment KEY for Friday, March 27:
Chapter Review, #1-2, 7-15

1. 2

7. It only applies to right triangles; false

8. true

9. \(c^2 = 5^2 + 5^2 = 25 + 25 = 50; \ c = \sqrt{50}\); false

10. \(\frac{BA}{ED} = \frac{AC}{DF}\)

11. \(\angle C \cong \angle F\)

12. \(\overline{EF}\)

13. 45°

14. \(2a = 16, a = 8; \ h = a \sqrt{3} = 8 \sqrt{3}\); the altitude is \(8 \sqrt{3}\) cm long.

15. \(a = 7, a \sqrt{2} = 7 \sqrt{2}\); the hypotenuse is \(7 \sqrt{2}\) cm long.
7th Grade Pre-Algebra  Chapter 10 Square Roots and Right Triangles

Square Roots
• Find the square root of perfect square numbers (You should have memorized.)
• Place a square root of a non-perfect square number between two integers
• Simplify and compare expressions involving square roots

Pythagorean Theorem
• Know the Pythagorean Theorem from memory
• Be capable of spelling Pythagorean
• Given two side lengths of a right triangle, solve for the third side
• Solve word problems using the Pythagorean theorem (See your worksheet for examples.)

Similar Triangles
• Given two similar triangles, find the missing side length(s) on one triangle
• Given two similar triangles, identify which sides are corresponding
• Given two similar triangles, write a proportion relating the side lengths
• Solve word problems using similar triangles
Pre-Algebra 7: Square Roots and Pythagorean Theorem
March 23-27

***Take on Wednesday March 25, 2020***

***NO Calculators***

Pre-Algebra Chapter 10 Quiz Name ___________________

1. Graph the following irrational numbers on the number line: \( \sqrt{60}, \sqrt{88}, \sqrt{19}, \sqrt{35}, \sqrt{8} \)

2. Simplify. If not a perfect square, put down which two integers it is between.
   2. \(- \sqrt{144}\)
   3. \(\sqrt{289} - \sqrt{100}\)
   4. \(- \sqrt{169} + 9\)
   5. \(\sqrt{229} - 4\)

6. Fill in <, >, or = to make a true statement.
   6. \(3\sqrt{25} \underline{\quad} \sqrt{3 \cdot 25}\)
   7. \(\sqrt{400} - \sqrt{300} \underline{\quad} \sqrt{400} - 300\)

8. Fill in the blanks. SPELLING COUNTS! The ____________________ states if the hypotenuse of a right triangle has length and legs with lengths and , then:

9. A right triangle has sides of lengths \(a, b,\) and \(c,\) with \(c\) the length of the hypotenuse. Find the length of the missing side.
   i. \(b = 96, \ c = 100\)
   ii. \(c = 20, \ a = 12\)
Assignment for Wednesday, March 25 (post-quiz!)

1. A tree 24 feet tall casts a shadow 12 feet long. Brad is 6 feet tall. How long is Brad's shadow? (draw a diagram and solve)

2. Triangles EFG and QRS are similar. The length of the sides of EFG are 144, 128, and 112. The length of the smallest side of QRS is 280, what is the length of the longest side of QRS?

3. A 40-foot flagpole casts a 25-foot shadow. Find the shadow cast by a nearby building 200 feet tall.

4. A girl 160 cm tall, stands 360 cm from a lamp post at night. Her shadow from the light is 90 cm long. How high is the lamp post?

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