## GreatHearts

## Euclidean Geometry

March 23-27

Time Allotment: 40 minutes per day

## Student Name:

## Teacher Name:

## Packet Overview

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## Additional Notes:

We are excited to review familiar concepts and dive into new material with all y'all. We hope you are all safe and have been washing your hands! We miss your smiling faces.

You will do sample problems for each lesson.
Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.

## Love,

Miss McCafferty and Mr. Bernstein

The answer key is on pg. 32. This should only be used when checking work.

## Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

## Monday, March 23

Geometry Unit: 3-D Figures
Lesson 1: Review Radicals

## Unit Overview: 3-D Figures

In our next unit in Euclidean Geometry we will be entering the world of 3-D figures! Before we can jump into 3-D Figures, though, we need to do some review of a few concepts you used in Pre-Algebra and Algebra.

This first week will on radicals and their rules. We need this review so that we can make sure that we are all on the same page. At the end of the week we will have a minor assessment so that I can make sure everyone is on board!

When we get to the week of March $30^{\text {th }}$ we will review plane figures and then jump into 3-D figures.

Objective: Be able to do this by the end of this lesson.

1. Simplify Radicals

## Lesson 1

## Review of Radicals \& Rules for Radicals

Square Roots:
Here are a few square roots:

$$
\sqrt{4}=2 \quad \sqrt{100}=10 \quad \sqrt{7} \approx 2.6457 \ldots
$$

The first two calculated cleanly (because they had perfect squares inside.) But, the last guy didn't. This is because it is an irrational number. Its decimal goes on forever and never repeats.

$$
\sqrt{7} \approx 2.6457 \ldots
$$

## Just like $\boldsymbol{\pi}$ :

The reason I used " ${ }^{\prime} \boldsymbol{*}$ " instead of a regular $"=$ " is because I can't write the exact number down.

## $\approx$ means $"$ approximately $=$ '

When roots like $\sqrt{7}$ don't calculate cleanly, in math, we just leave them as radicals. It's more accurate this way!

$$
\text { So... } \sqrt{5} \text { ? Just leave it as } \sqrt{5} \text { ! Done! }
$$

Simplify the following: (write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)
1.
2.


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## Cube Roots:

Here are a few cube roots:

$$
\sqrt[3]{8}=2 \quad \sqrt[3]{1}=1 \quad \sqrt[3]{-64}=-4
$$

OK... So, how am I figuring these out?
Pretty easy.
If I want $\sqrt[3]{27}$, I ask myself
"What number cubed gives me 27?'"

$$
(3)^{3}=27 \xrightarrow{()^{3}} \sqrt[3]{27}=3
$$

Here's a trickier one:

$$
\sqrt[3]{-8}=?
$$

Hey, that number under the radical is negative! Are we allowed to do this?

## Let's think about it...

Can we find a number that goes in this?

$$
(\underset{\uparrow}{ })^{3}=-8
$$

Absolutely!

$$
\begin{aligned}
(-2)^{3}= & (-2)(-2)(-2)=-8 \\
& \text { so } \sqrt[3]{-8}=-2
\end{aligned}
$$

Now that we're thinking like this, let's go back and look at square roots...

$$
\sqrt{36}=?
$$

Can we find a number that goes in here?

$$
\begin{gathered}
()^{2}=(\underset{\uparrow}{ })\left(\begin{array}{r}
) \\
(6)^{2}
\end{array}=36\right)(6)=36 \\
\text { so } \sqrt{36}=6
\end{gathered}
$$

By the way, this radical sign is really

$$
\sqrt[2]{36}=6
$$

But, square roots are so commonly used, that we just leave off the $\mathbf{2}$ and assume it's there.

What about $\sqrt{-49}$.
Can we find a number that goes in this?

$$
()_{\uparrow}^{2}=(\underset{N}{ })(\underset{N}{ })=-49
$$

Since these are the same number, wed either have

$$
\begin{gathered}
(-)(-)=+ \text { or }(t)(t)=+ \\
()^{2}=-49 \\
\text { That's why a square can't be negative! }
\end{gathered}
$$

In fact, this goes for ALL even roots.

$$
\sqrt[4]{-16} \longleftarrow \text { Can't do! }
$$

Since no number works in

$$
()^{4}=-16
$$

There'd be an even number of this guy multiplied together:

$$
\left.\begin{array}{rl}
())()() & =-16 \\
(-)(-)(-)(-) & =+M \\
(+)(+)(+)(+) & =+
\end{array}\right\}
$$

This negative can't happen!
I would like to thank the awesome people at CoolMath.com for their wonderful resources.
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OK, now here's something we have to get really picky about:

$$
\sqrt{x^{2}}=?
$$

What goes in here?


Is it just x ?
The answer is... sometimes.

## Check it out:

## What if $x=3$ ?

$$
\sqrt{(3)^{2}}=\sqrt{9}=3
$$

Yeah, that works.

## What if $\mathrm{x}=\mathbf{- 3}$ ?

$$
\begin{gathered}
\sqrt{(-3)^{2}}=\sqrt{9}=3 \\
\uparrow \\
\text { Not the same! }
\end{gathered}
$$

So, it doesn't work when $x$ is negative!

Technically,

$$
\sqrt{x^{2}}=x \text { only when } x \geq 0
$$

If $X<\boldsymbol{O}$, we'd say

$$
\sqrt{x^{2}}=|x|
$$

Look again at our example:

$$
\sqrt{(-3)^{2}}=|-3|=3
$$

Yep, it works!
Here's the rule for multiplying radicals:


* Note that the types of root, $n$, have to match!

Here are a few examples of multiplying radicals:

$$
\sqrt{2} \sqrt{8}=\sqrt{2 \cdot 8}=\sqrt{16}=4
$$

Pop these into your calculator to check!

$$
\sqrt[3]{2} \sqrt[3]{4}=\sqrt[3]{2 \cdot 4}=\sqrt[3]{8}=2
$$

# $\sqrt{5} \sqrt{7}=\sqrt{5 \cdot 7}=\sqrt{35} \leftarrow$ Can't pop: 

$$
\sqrt{3} \sqrt{3}=\sqrt{3 \cdot 3}=\sqrt{9}=3
$$

$2 \sqrt{2} \cdot 3 \sqrt{50}$ Do a little commuting... $=2 \cdot 3 \cdot \sqrt{2} \sqrt{50}=6 \sqrt{2 \cdot 50}$ $=6 \sqrt{100}=6 \cdot 10=60$

Simplify the following: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

1
$\sqrt[4]{3} \sqrt[4]{27}$

2
$\sqrt{6} \sqrt{6}$

## Here's the rule for dividing radicals:


*Note again that the types of roots, $n$, need to match!
Here are a few examples:

$$
\frac{\sqrt{32}}{\sqrt{2}}=\sqrt{\frac{32}{2}}=\sqrt{16}=4
$$

## Check it on your calculator!

$$
\frac{\sqrt[3]{3}}{\sqrt[3]{81}}=\sqrt[3]{\frac{3}{81}}=\sqrt[3]{\frac{1}{27}}=\frac{\sqrt[3]{1}}{\sqrt[3]{27}}=\frac{1}{3}
$$

(We used the rule twice here!)


Here is another example:

$$
\begin{aligned}
\frac{12 \sqrt{32}}{2 \sqrt{8}}=\frac{12}{2} \sqrt{\frac{32}{8}} & =6 \sqrt{4} \\
& =6 \cdot 2=12
\end{aligned}
$$

## Simplify the following: (Continue to use the same sheet of paper if you can. Show your work.)

1

2


I know all of this is review, but it is important that we go back over this and revisit these concepts before we can move on.

Thank you for all your hard work. That's all for today!

## Tuesday, March 24

Welcome back! We still have review to do so here we go!
Objective: Be able to do this by the end of this lesson.

1. Add Radicals

## Geometry Unit: 3-D Figures

Lesson 2: Review Radicals: Simplifying \& Adding Radicals

Before calculators, mathematicians had to crunch out decimals for things like

by hand using long division!
They had tables that listed decimal values for radicals:
Table 1

| $n$ | $n^{2}$ | $\sqrt{n}$ | $n^{3}$ | $\sqrt[3]{n}$ |
| :---: | :---: | :---: | ---: | ---: |
| 1 | 1 | 1.000 | 1 | 1.000 |
| 2 | 4 | 1.414 | 8 | 1.260 |
| 3 | 9 | 1.732 | 27 | 1.442 |
| 4 | 16 | 2.000 | 64 | 1.587 |
| 5 | 25 | 2.236 | 125 | 1.710 |
| 6 | 36 | 2.449 | 216 | 1.817 |
| 7 | 49 | 2.646 | 343 | 1.913 |
| 8 | 64 | 2.828 | 512 | 2.000 |
| 9 | 81 | 3.000 | 729 | 2.080 |

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And they had some basic ones memorized like

$$
\sqrt{2} \approx 1.414
$$

So, if they needed a decimal value for

$$
7 \sqrt{2}
$$

they'd put in the memorized $\mathbf{1 . 4 1 4}$ part and crunch it out by hand:

$$
7 \sqrt{2} \approx 7(1.414) \longrightarrow \begin{aligned}
& 1.414 \\
& \frac{\times 7}{9.898}
\end{aligned}
$$

Not very accurate either. My calculator says

$$
7 \sqrt{2} \approx 9.89949 \ldots
$$

So... What if they needed to crunch this?

$$
5 \sqrt{200}
$$

Well... $\sqrt{200}$ is too big for the tables...
So, what did they do?
They rewrote the radical so that the smallest number possible was inside the radical.

Check it out:

$$
\begin{aligned}
5 \sqrt{200}=5 \sqrt{100 \cdot 2}=5 \sqrt{100} \sqrt{2} & =5 \cdot 10 \sqrt{2} \\
& =50 \sqrt{2}
\end{aligned}
$$

Now, since they knew that $\sqrt{2} \approx 1.414$, they could calculate a fairly accurate decimal. BUT, we're going to just leave it in its pure form since it's more accurate this way!

What we're really doing when we try to rewrite these things is that we are going on a perfect square hunt!

Look at this guy:


See any perfect squares hiding in here?
What about 25?

$$
\sqrt{75}=\sqrt{25 \cdot 3}=\sqrt{25} \sqrt{3}=5 \sqrt{3}
$$

SIMPLIFY: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

## 1. Rewrite $\sqrt{45}$ so that the smallest number possible is inside the radical.

Here's a trickier one:

$$
\sqrt{588}
$$

There are two perfect squares hiding in here...
We'll need to completely factor this thing to find them.

$$
\begin{aligned}
\sqrt{588}=\sqrt{4 \cdot 49 \cdot 3} & =\sqrt{4} \sqrt{49} \sqrt{3} \\
& =2 \cdot 7 \sqrt{3}=14 \sqrt{3}
\end{aligned}
$$

*You can check to see if this is right by
popping $\sqrt{588}$ and $14 \sqrt{3}$ into your calculator... You should get the same thing.

SIMPLIFY: (write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

## 2. Rewrite $\sqrt{1575}$ so that the smallest number possible is inside the radical.

You can do this with other kinds of roots too -- like cube roots.

## Check it out:

We'll be on a perfect cube hunt this time!

$$
\sqrt[3]{56}=\sqrt[3]{8 \cdot 7}=\sqrt[3]{8} \sqrt[3]{7}=2 \sqrt[3]{7}
$$

Simplify: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

## 3. Rewrite $\sqrt[3]{54}$ so that the smallest number possible is inside the radical.

## Adding Radicals:

We want to add these guys without using decimals:

$$
\sqrt{2}+\sqrt{8}
$$

The method to adding radicals is to simplify everything and see if we can combine anything.

$$
\begin{gathered}
\sqrt{2} \text { is already done. } \\
\sqrt{8}=\sqrt{4 \cdot 2}=\sqrt{4} \sqrt{2}=2 \sqrt{2} \\
\text { So, } \sqrt{2}+\sqrt{8}=\sqrt{2}+2 \sqrt{2}
\end{gathered}
$$

Now, we treat the radicals like variables. Think of it as

$$
\begin{gathered}
y+2 y=3 y \\
\sqrt{2}+\sqrt{8}=\sqrt{2}+2 \sqrt{2}=3 \sqrt{2}
\end{gathered}
$$

Done!

Here's another one:

$$
\sqrt{48}-\sqrt{3}+\sqrt{75}
$$

Rewrite the radicals...

$$
\begin{aligned}
\sqrt{48}-\sqrt{3}+\sqrt{75} & =\sqrt{16 \cdot 3}-\sqrt{3}+\sqrt{25 \cdot 3} \\
& =4 \sqrt{3}-\sqrt{3}+5 \sqrt{3} \\
& =8 \sqrt{3}
\end{aligned}
$$

(Do it like $4 \mathrm{x}-\mathrm{x}+5 \mathrm{x}=8 \mathrm{x}$.)

## SIMPLIFY: (Do your work on the same sheet of paper you used for the previous lesson 2 problems. Show your work)

$$
\text { 4. } \sqrt{45}+\sqrt{20}+\sqrt{5}-\sqrt{125}
$$

I hope this review is helpful in highlighting what you have down and what you needed to go back and review!

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## Wednesday, March 25

Objective: Be able to do this by the end of this lesson.

1. Multiply Radicals
2. Rationalize Denominators

Review of Radicals: Multiplying Radicals \& Rationalizing Denominators
We're going to need this for the next lesson.
We'll need to multiply guys like

$$
(7-\sqrt{2})(5+\sqrt{3})
$$

There's an easy process we can use called FOIL.

$$
\begin{aligned}
& \mathbf{F}=\text { First } \\
& \mathbf{O}=\text { Outer } \\
& \mathbf{I}=\mathbf{I n n e r} \\
& \mathbf{L}=\mathbf{L a s t}
\end{aligned}
$$

You can use this process anytime you multiply two variables/numbers that are added or subtracted inside of two sets of parentheses. Like this:

$$
(a+b)(c+d)
$$



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OK, now let's do it with some radicals:

$$
\begin{aligned}
& (7-\sqrt{2})(5+\sqrt{3}) \\
& (7-\sqrt{2})(5+\sqrt{3}) \\
& =(7)(5)+7 \sqrt{3}-5 \sqrt{2}-\sqrt{2} \sqrt{3} \\
& =35+7 \sqrt{3}-5 \sqrt{2}-\sqrt{6}
\end{aligned}
$$

We have to stop here since we can't combine anything!

SIMPLIFY: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

1

$$
(9+\sqrt{2})(6-\sqrt{5})
$$

Here's another one -- and something cool is going to happen:

$$
\begin{gathered}
(5+\sqrt{2})(5-\sqrt{2}) \\
(5+\sqrt{2})(5-\sqrt{2}) \\
=(5)(5)-5 \sqrt{2}+5 \sqrt{2}-\sqrt{2} \sqrt{2} \\
=25-\sqrt{4}=25-2=23
\end{gathered}
$$

No radicals in the answer!

SIMPLIFY: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

2

$$
(3+\sqrt{7})(3-\sqrt{7})
$$

## Rationalizing Denominators

Mathematicians used to calculate decimals for problems like this

things got really ugly.
They'd have to use a table to get the value for $\sqrt{2}$... Then, they'd have to do the long division by hand:

$$
\sqrt { 2 } = 1 . 4 1 4 \longrightarrow 1 . 4 1 4 \longdiv { 3 . 0 0 0 }
$$

So, it was a LOT easier for them if there wasn't a radical in the denominator.
This is where they brought in "rationalizing the denominator."
Here's how it goes:


Here's another one:


SIMPLIFY: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

## Rationalize the denominator:

3


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They get messier...
What if we needed to get the radical out of this denominator?


Same game -- we still multiply by a fancy version of $1 .$.


The fancy version of 1 will be made up of this guy's conjugate.
Well now, that was helpful wasn't it?
Conjugate $=$ same value, different sign

So, the conjugate of

$$
2-\sqrt{3} \text { is } 2+\sqrt{3}
$$

Check out how it works:


Put () around everyone or you're really going to mess things up!
distribute


Not only did we eliminate the radical in the denominator, in this one, we eliminated the whole denominator.

Here's another one:


$$
\begin{aligned}
& \frac{4}{1+\sqrt{7}}=\frac{4}{(1+\sqrt{7})} \cdot \frac{(1-\sqrt{7})}{(1-\sqrt{7})}=\frac{4-4 \sqrt{7}}{1-\sqrt{49}} \\
& =\frac{4-4 \sqrt{7}}{1-7}=\frac{4-4 \sqrt{7}}{-6}=\frac{-2(-2+2 \sqrt{7})}{-6} \\
& =\frac{-2+2 \sqrt{7}}{3}
\end{aligned}
$$

SIMPLIFY: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

## Rationalize the denominator:

4

$$
\frac{3}{2-\sqrt{6}}
$$

Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly. Make sure you are showing all of your work.

Thursday, March 26
Objective: Be able to do this by the end of this lesson.

1. Simplify and alternate notation for Fractional exponents,

## Review of Radicals: Fractional Exponents

Don't get all nervous about these -- it's just a different notation for what you've already been doing.

$$
\begin{aligned}
& \sqrt[3]{8} \text { can be written as } 8^{1 / 3} \\
& \sqrt{15} \text { can be written as } 15^{1 / 2}
\end{aligned}
$$

* Remember that $\sqrt{\square}$ is really $\sqrt[2]{ }$... We just assume the 2.

Not that bad, is it?
Here's the general rule:


This notation has some advantages because it will help you understand some things better.

Check it out:

$$
\sqrt{5} \cdot \sqrt{5}=5^{1 / 2} \cdot 5^{1 / 2}=5^{1 / 2+1 / 2}=5^{1}=5
$$

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Now, we can show why this rule works:


We can do the same thing with this rule:


We can also figure out weird numbers like


Using this rule:


The only tricky part on this one is to remember how to write the fraction:


## It's alphabetical!

$$
16^{3 / 4}=\sqrt[4]{16^{3}}
$$

By the way, there's kind of a cool way to calculate this without a calculator.
$\begin{gathered}\text { You can rewrite } \\ \text { this guy as a } \\ \text { 4th power. }\end{gathered}$$\rightarrow 16^{3 / 4}$


TRY IT: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

Calculate this without a calculator:

1. $9^{\frac{5}{2}}$

You can use this new notation to help you deal with messy radical problems.
Check it out:


Here's another one:

$$
\begin{aligned}
\sqrt[3]{8 x^{3} w^{9} z^{6}} & =\left(8 x^{3} w^{9} z^{6}\right)^{1 / 3} \\
& =8^{1 / 3}\left(x^{3}\right)^{1 / 3}\left(w^{9}\right)^{1 / 3}\left(z^{6}\right)^{1 / 3} \\
& =2 x w^{3} z^{2}
\end{aligned}
$$

TRY IT: (Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.)

1


2


## Euclidean Geometry

March 23-27

## GreatHearts

## Friday, March 27

Objective: Be able to do this by the end of this lesson.

1. Take Minor Assessment on Radicals

## Minor Assessment: Radicals

Answers should be left in simplest radical form. There should not be any decimals in your answers. Write out your answers on a separate sheet of paper and show your work. Make sure your answers are written out on a sheet of paper with the date and the lesson written clearly at the top. Make sure you are showing all of your work.

1. Simplify

$$
3 \sqrt{900}
$$

6. Simplify

$$
(5 \sqrt{2 x}+\sqrt{5})(-4 \sqrt{2 x}+\sqrt{5 x})
$$

2. Simplify

$$
3 \sqrt{8}+2 \sqrt{27}+3 \sqrt{3}
$$

7. Simplify

$$
\frac{\sqrt{5}+2 \sqrt{2}}{4-\sqrt{5}}
$$

3. Simplify

$$
\sqrt{15}(\sqrt{3}+\sqrt{10})
$$

8. Simplify

$$
\sqrt{384 x^{4} y^{3}}
$$

4. Simplify

$$
\frac{4}{3 \sqrt{5}}
$$

9. Simplify

$$
\sqrt[3]{-16 a^{3} b^{8}}
$$

5. Simplify

$$
-3 \sqrt{7 r^{3}} \cdot 6 \sqrt{7 r^{2}}
$$

10. Simplify

$$
\frac{\sqrt[3]{10}}{\sqrt[3]{625}}
$$

Answer Key

Monday, March 23
Pg. 3

1. 3
2. $\sqrt{ } 37$

Pg. 9

1. 3
2. 6

Pg. 11

1. $1 / 10$
2. 5
3. 15

Tuesday, March 24
Pg. 14

1. $3 \sqrt{ } 5$

Pg. 15
2. $15 \sqrt{ } 7$

Pg. 16
3. $3 \sqrt[3]{2}$

Pg. 17
4. $\sqrt{ } 5$

Wednesday, March 25
Pg. 19

1. $54-9 \sqrt{5}+6 \sqrt{2}-\sqrt{10}$

Pg. 20
2. 2

Pg. 22
3. $\frac{2 \sqrt{15}}{5}$

Pg. 25
4. $\frac{6+3 \sqrt{6}}{-2}$ or $\frac{-6+-3 \sqrt{6}}{2}$

Thursday, March 26
Pg. 29

1. 243

Pg. 30

1. $\frac{a^{2} b^{4}}{4}$

Friday, March 27
Minor Assessment Answers

1. 90
2. $6 \sqrt{2}+9 \sqrt{3}$
3. $3 \sqrt{5}+5 \sqrt{6}$
4. $\frac{4 \sqrt{5}}{15}$
5. $-126 r^{2} \sqrt{r}$
6. $-40 x+5 x \sqrt{10}-4 \sqrt{10 x}+$
$5 \sqrt{x}$
7. $\frac{4 \sqrt{5}+5+8 \sqrt{2}+2 \sqrt{10}}{11}$
8. $8 x^{2} \cdot y \sqrt{6 y}$
9. $-2 a b^{2} \sqrt[3]{2 b^{2}}$
10. $\frac{\sqrt[3]{2}}{5}$

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