GreatHearts®

Pre-Calculus: Week of April 13 - 17

Time Allotment: 40 minutes per day

Student Name: ________________________________

Teacher Name: Mrs. Melisa R. Walters

Pre-Calculus: Chapter 9 Systems of Equations and Inequalities
April 6 – 10, 2020

Packet Overview

Date	Objective(s)	Page Number
Monday, April 13	*** HOLIDAY ***	2
Tuesday, April 14	Evaluate a system of linear equations using Matrices	2
Wednesday, April 15	Calculate matrix operations to solve systems with Inverses	5
Thursday, April 16	Identify Systems with Inverses	9
Friday, April 17	Evaluate a System of linear equations Using the Inverse of a Matrix	13

Additional Notes: We miss you all very much!

I hope that you are all staying safe and keeping that positive attitude that you always bring with you to class. Any questions? Email me directly: melisa.walters@greatheartsnorthernoaks.org

Each lesson will end with a set of math problems pertaining to that particular lesson of the day.

Please create an "Exercise Packet" which is to include all your work and completion of these daily exercises. Each day is to have a title with the date followed by the name of the lesson. Please include a title page and staple all the completed exercises. And starting this week we will have "Office Hours" via Zoom!

Period 3 Monday and Wednesday from 1:00PM – 1:50 PM
Period 4 Tuesday and Thursday from 10:00AM – 10:50 AM
Period 6 Tuesday and Thursday from 1:00PM – 1:50 PM

This means during the designated times only, you can contact us to ask questions, go over problems etc… For example, if you had Pre-Calculus during Period 3, then on Monday and Wednesday from 1-1:50 you can connect via Zoom and not any other time. More details to follow on specific links, access codes and etiquette. Note, Quiz this week on Thursday. It is located at the back of the packet.

Thank you for your hard work students. I appreciate all of you. Have a great day!

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

____

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

____

Monday, April 13

*** HOLIDAY ***

Tuesday, April 14

Pre-Calculus: chapter 9
Lesson 2: Solve a system of linear equations using Matrices

Objective: Evaluate a system of linear equations using Matrices

NOTE: I will be providing some examples specific to this lesson. Please highlight the key points as you read through this entire lesson or take notes in the margin. At the end of the lesson, the list of questions in the exercise are to be completed to ensure mastery.

Lesson 2
RECAP:

$$\begin{array}{ccc}
 & \overrightarrow{\quad n \text{ columns} \quad} & \\
 & \begin{array}{cccccc} \text{Column 1} & \text{Column 2} & & \text{Column } j & & \text{Column } n \\ \downarrow & \downarrow & & \downarrow & & \downarrow \end{array} & \\
A=[a_{ij}]_{m\times n} = & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix} & \begin{array}{l} \leftarrow \text{Row 1} \\ \leftarrow \text{Row 2} \\ \\ \leftarrow \text{Row } i \\ \\ \leftarrow \text{Row } m \end{array} \Bigg\downarrow \; m \text{ rows}
\end{array}$$

Each number [a_{ij}] of the matrix has two indexes; the **row index** i and the **column index** j
The matrix shown in display above has m rows and n columns. The numbers a_{ij} are usually referred to as the **entries** of the matrix. For example, a_{23} refers to the entry in the second row, third column.

We have covered a lot of material over the past three weeks and the next lesson presents an opportunity to ensure you have learned the concepts on matrices we studied. Please answer all the questions below and then apply those answers to color in the picture. Enjoy the beauty of mathematics.

GreatHearts®

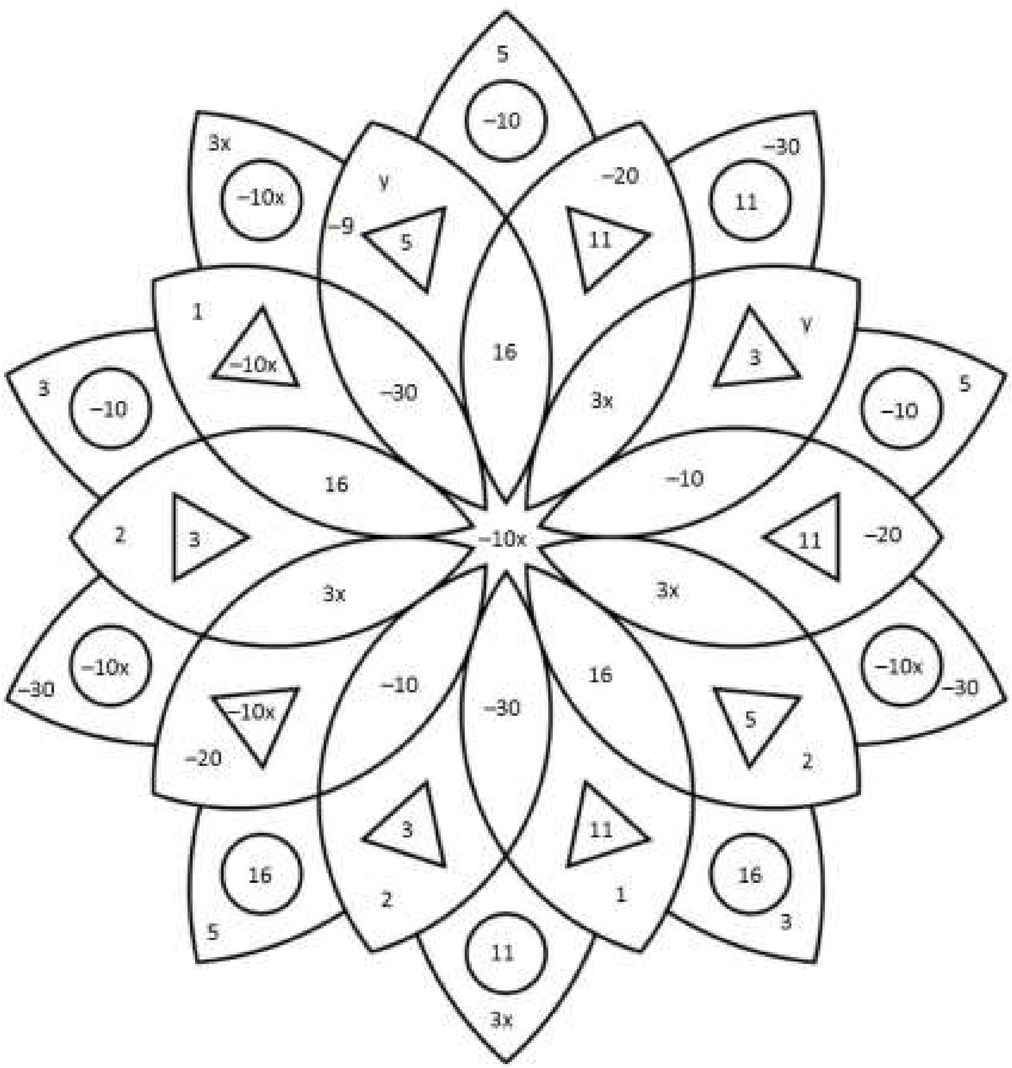
Basic Matrix Operations Color by Number

Directions: Simplify each matrix below. Then, color the given specific entry on the picture with the indicated color

Matrix	Solution	Entry
1. $\begin{bmatrix} 1 & 0 \\ 2 & -3 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -5 \\ -1 & 1 \\ -4 & -5 \end{bmatrix}$	______________	a_{12} pink
2. $-2\begin{bmatrix} 5 & 15 & -2 \\ 3 & -2 & -1 \end{bmatrix}$	______________	a_{12} purple
3. $[5 \quad 2] + [-3 \quad -7]$	______________	a_{11} green
4. $-5[3 \quad -4 \quad 0 \quad 2]$	______________	a_{14} blue
5. $-2x[4x \quad y^2 \quad -7x \quad 5]$	______________	a_{14} yellow
6. $10\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$	______________	a_{31} black
7. $[2 \quad -7 \quad -6] + [1 \quad -8 \quad -1]$	______________	a_{11} pink
8. $3\begin{bmatrix} 3x \\ 2y^2 \\ x \end{bmatrix}$	______________	a_{31} purple
9. $\begin{bmatrix} -6x + y \\ -x \\ 6z \end{bmatrix} + \begin{bmatrix} 6x \\ 3y \\ 2 - x \end{bmatrix}$	______________	a_{11} green
10. $5[3 \quad 2 \quad 2 \quad -7] - [2 \quad 4 \quad -6 \quad 2]$	______________	a_{13} blue
11. $\begin{bmatrix} 4 & 3 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} -6 & 7 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} -3 & 4 \\ 1 & -6 \end{bmatrix}$	______________	a_{22} yellow
12. $\begin{bmatrix} 2 & 3 \\ -4 & -4 \\ 6 & 8 \end{bmatrix} + 5\begin{bmatrix} -9 & 3 \\ 6 & 1 \\ -4 & -3 \end{bmatrix}$	______________	a_{22} black

GreatHearts®

Name____Date________________________



Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Exercises for Tuesday, April 14, 2020
Complete the Color by Number assignment and you may give your parents a flower(s).
Yes, Mrs. Walters would like to receive this too as a part of the homework packet assignment. I look forward to receiving my flowers!!

Wednesday, April 15, 2020

Pre-Calculus: chapter 9

Lesson 3: Solve systems with Inverses

Objective: Calculate matrix operations to solve systems with Inverses

Lesson 3:
Now that you have done many operations on matrices, let's go over the different types of matrices.

Row Matrix: A matrix which is having only one row is called row matrix.
Examples:
This is a 1 x 4

$$\begin{bmatrix} 5 & -2 & 7 & 9 \end{bmatrix}$$

This row matrx is a 1 x 3:

$$\begin{bmatrix} a & b & c \end{bmatrix}$$

Column Matrix: A matrix which is having only one column.

Example: 3 x 1 is

$$\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

Square Matrix
A square matrix is a matrix in which the number of rows and the number of columns are equal. A matrix of order n x n is also known as a square matrix of order n.

In a square matrix, A of the order n x n the elements $a_{11}, a_{22}, a_{33}........a_{nn}$ is called principal diagonal or leading diagonal or **main diagonal elements**.
This is an example of a 3 x 3 matrix:

$$\begin{bmatrix} 3 & 2 & 1 \\ -5 & 0 & 11 \\ -3 & 2 & 8 \end{bmatrix}$$

Diagonal Matrix: In a diagonal matrix all the entries except the entries along the main diagonal are zero.

Example: $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Triangular Matrix
A square matrix is known as a lower triangular matrix if all elements above main diagonal are zero.

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 5 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

A square matrix is known as an upper triangular matrix if all elements below the diagonal are zero.

$$\begin{bmatrix} 3 & 0 & 0 \\ 5 & 8 & 0 \\ 1 & -2 & 5 \end{bmatrix}$$

Scalar Matrix
A square matrix is known as a scalar matrix if all the entries along the main diagonal are equal.

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Identity or scalar Matrix
When all the entries along the main diagonal are equal to 1 is known as identity matrix or unit matrix. Usually identity matrix is denoted by I_2 if the order of that particular matrix is 2 x 2 and
it is denoted by I_3 if the order of that particular matrix is 3 x 3.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

GreatHearts®

Example 1 **Showing That the Identity Matrix Acts as a 1**

Given matrix A, show that $AI = IA = A$.

$$A = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$

Solution Use matrix multiplication to show that the product of A and the identity is equal to the product of the identity and A.

$$AI = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 + 4 \cdot 0 & 3 \cdot 0 + 4 \cdot 1 \\ -2 \cdot 1 + 5 \cdot 0 & -2 \cdot 0 + 5 \cdot 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$

$$AI = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 \cdot 3 + 0 \cdot (-2) & 1 \cdot 4 + 0 \cdot 5 \\ 0 \cdot 3 + 1 \cdot (-2) & 0 \cdot 4 + 1 \cdot 5 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$

STEPS on how: Given two matrices, show how one is the multiplicative inverse of the other.

1. Given matrix A of the order n x n and matrix B of order n x n multiply AB.
2. If AB = I, then find the product BA. If BA = I, then $B = A^{-1} and\ A = B^{-1}$.

Example 2 **Showing That Matrix *A* Is the Multiplicative Inverse of Matrix *B***

Show that the given matrices are multiplicative inverses of each other.

$$A = \begin{bmatrix} 1 & 5 \\ -2 & -9 \end{bmatrix}, B = \begin{bmatrix} -9 & -5 \\ 2 & 1 \end{bmatrix}$$

Solution Multiply AB and BA. If both products equal the identity, then the two matrices are inverses of each other.

$$AB = \begin{bmatrix} 1 & 5 \\ -2 & -9 \end{bmatrix} \begin{bmatrix} -9 & -5 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-9) + 5(2) & 1(-5) + 5(1) \\ -2(-9) - 9(2) & -2(-5) - 9(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -9 & -5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ -2 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -9(1) - 5(-2) & -9(5) - 5(-9) \\ 2(1) + 1(-2) & 2(-5) + 1(-9) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A and B are inverses of each other.

<u>Zero matrix (or) null matrix (or) void matrix</u>: In a matrix, if all the entries are zero then it is called zero matrix.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

<u>Transpose of matrix</u>

A matrix formed by interchanging rows as columns and columns as rows is called as transpose of a matrix. The transpose of matrix A is usually denoted by A^T.

If $A =$

$$\begin{bmatrix} 1 & 5 & 7 \\ 2 & 3 & 8 \end{bmatrix}$$

then $A^T =$

$$\begin{bmatrix} 1 & 2 \\ 5 & 3 \\ 7 & 8 \end{bmatrix}$$

Exercise for Wednesday April 15, 2020

Show that the following two matrices are inverses of each other.

$$A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}, B = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

Thursday, April 16

Pre-Calculus: chapter 9
Lesson 4: Solving Systems with Inverses

****NOTE: First, please complete the quiz located at the end of the packet. Then, you may begin this lesson.*

Objective: Solving Systems with Inverses continued

Lesson 4: Finding the Multiplicative Inverse by Augmenting with the Identity
Another way to find the multiplicative inverse is by augmenting with the identity. When matrix A is transformed into I, the augmented matrix I transforms into A^{-1}.

For example, given

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$$

Perform row operations with the goal of turning A into the identity.

1. Switch row 1 and row 2.

$$\left[\begin{array}{cc|cc} 5 & 3 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}\right]$$

2. Multiply row 2 by -2 and add to row 1.

$$\left[\begin{array}{cc|cc} 1 & 1 & -2 & 1 \\ 2 & 1 & 1 & 0 \end{array}\right]$$

3. Multiply row 1 by -2 and add to row 2.

$$\left[\begin{array}{cc|cc} 1 & 1 & -2 & 1 \\ 0 & -1 & 5 & -2 \end{array}\right]$$

4. Add row 2 to row 1.

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & -1 & 5 & -2 \end{array}\right]$$

5. Multiply row 2 by -1.

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

The matrix we have found is A^{-1}.

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

Finding the Multiplicative Inverse Using Matrix Multiplication
We can now determine whether two matrices are inverses, but how would we find the inverse of a given matrix? Since we know that the product of a matrix and its inverse is the identity matrix, we can find the inverse of a matrix by setting up an equation using matrix multiplication.

Example: Finding the Multiplicative Inverse Using Matrix Multiplication
Use matrix multiplication to find the inverse of the given matrix.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

Solution: For this method, we multiply A by a matrix containing unknown constants and set it equal to the identity.

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find the product of the two matrices on the left side of the equal sign

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1a - 2c & 1b - 2d \\ 2a - 3c & 2b - 3d \end{bmatrix}$$

Next, set up a system of equations with the entry in row 1, column 1 of the new matrix equal to the first entry of the identity, 1. Set the entry in row 2, column 1 of the new matrix equal to the corresponding entry of the identity, which is 0.

$$1a - 2c = 1 \qquad R_1$$
$$2a - 3c = 0 \qquad R_2$$

Using row operations, multiply and add as follows: $(-2)\,R_1 + R_2 \rightarrow R_2$.
Add the equations, and solve for c.

$$1a - 2c = 1$$
$$0 + 1c = -2$$
$$c = -2$$

Back-substitute to solve for a.

$$a - 2(-2) = 1$$
$$a + 4 = 1$$
$$a = -3$$

Write another system of equations setting the entry in row 1, column 2 of the new matrix equal to the corresponding entry of the identity, 0. Set the entry in row 2, column 2 equal to the corresponding entry of the identity.

$$1b - 2d = 0 \qquad R_1$$
$$2b - 3d = 1 \qquad R_2$$

Using row operations, multiply and add as follows: $(-2)\,R_1 + R_2 = R_2$.
Add the two equations and solve for d.

$$\begin{aligned} 1b - 2d &= 0 \\ 0 + 1d &= 1 \\ \hline d &= 1 \end{aligned}$$

Once more, back-substitute and solve for b.

$$\begin{aligned} b - 2(1) &= 0 \\ b - 2 &= 0 \\ b &= 2 \end{aligned}$$

$$A^{-1} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

<u>Finding the Multiplicative Inverse of 2 × 2 Matrices Using a Formula</u>
When we need to find the multiplicative inverse of a 2 × 2 matrix, we can use a special formula instead of using matrix multiplication or augmenting with the identity.

If A is a 2×2 matrix, such as

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the multiplicative inverse of A is given by the formula

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

where $ad - bc \neq 0$. If $ad - bc = 0$, then A has no inverse.

Example: Using the Formula to Find the Multiplicative Inverse of Matrix A
Use the formula to find the multiplicative inverse of

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

Solution: Using the formula, we have

$$A^{-1} = \frac{1}{(1)(-3)-(-2)(2)}\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$
$$= \frac{1}{-3+4}\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

Analysis We can check that our formula works by using one of the other methods to calculate the inverse. Let's augment A with the identity.

Perform row operations with the goal of turning A into the identity.

1. Multiply row 1 by −2 and add to row 2.

$$\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

2. Multiply row 1 by 2 and add to row 1.

$$\left[\begin{array}{cc|cc} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

So, we have verified our original solution.

$$A^{-1} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

Example 5 Finding the Inverse of the Matrix, If It Exists

Find the inverse, if it exists, of the given matrix.

$$A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$$

Solution We will use the method of augmenting with the identity.

$$\left[\begin{array}{cc|cc} 3 & 6 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$$

1. Switch row 1 and row 2.

$$\left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 3 & 6 & 1 & 0 \end{array}\right]$$

2. Multiply row 1 by −3 and add it to row 2.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{array}\right]$$

3. There is nothing further we can do. The zeros in row 2 indicate that this matrix has no inverse.

Finding the Multiplicative Inverse of 3 × 3 Matrices

Unfortunately, we do not have a formula similar to the one for a 2 × 2 matrix to find the inverse of a 3 × 3 matrix. Instead, we will augment the original matrix with the identity matrix and use row operations to obtain the inverse.

Given a 3 × 3 matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

augment A with the identity matrix

$$A\,|\,I = \left[\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 & 1 & 0 \\ 2 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

To begin, we write the augmented matrix with the identity on the right and A on the left. Performing elementary row operations so that the identity matrix appears on the left, we will obtain the inverse matrix on the right. We will find the inverse of this matrix in the next example.

Given a 3 × 3 matrix, find the inverse

1. Write the original matrix augmented with the identity matrix on the right.
2. Use elementary row operations so that the identity appears on the left.
3. What is obtained on the right is the inverse of the original matrix.
4. Use matrix multiplication to show that $AA^{-1} = I$ and $A^{-1}A = I$.

Exercise for Thursday, April 16, 2020
9.7 Section Exercise page 840 # 13, 15 extra credit complete #21

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Friday, April 17, 2020

Pre-Calculus: chapter 9
Lesson 5: Solving a System of linear equations Using the Inverse of a Matrix

Objective: Evaluate a System of linear equations Using the Inverse of a Matrix

Lesson
Solving a system of linear equations using the inverse of a matrix requires the definition of two new matrices: X is the matrix representing the variables of the system, and B is the matrix representing the constants. Using matrix multiplication, we may define a system of equations with the same number of equations as variables as AX =B.

To solve a system of linear equations using an inverse matrix, let A be the coefficient matrix, let X be the variable matrix, and let B be the constant matrix. Thus, we want to solve a system AX = B. For example, look at the following system of equations.

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

From this system, the coefficient matrix is

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

The variable matrix is

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

And the constant matrix is

$$B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Then AX = B looks like

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Recall the discussion earlier in this section regarding multiplying a real number by its inverse, $(2^{-1})\,2 = \left(\frac{1}{2}\right)2 = 1$. To solve a single linear equation $ax = b$ for x, we would simply multiply both sides of the equation by the multiplicative inverse (reciprocal) of a. Thus,

$$\begin{aligned} ax &= b \\ \left(\frac{1}{a}\right)ax &= \left(\frac{1}{a}\right)b \\ (a^{-1})ax &= (a^{-1})b \\ [(a^{-1})a]x &= (a^{-1})b \\ 1x &= (a^{-1})b \\ x &= (a^{-1})b \end{aligned}$$

The only difference between solving a linear equation and a system of equations written in matrix form is that finding the inverse of a matrix is more complicated, and matrix multiplication is a longer process. However, the goal is the same—to isolate the variable. We will investigate this idea in detail, but it is helpful to begin with a 2 × 2 system and then move on to a 3 × 3 system.

solving a system of equations using the inverse of a matrix

Given a system of equations, write the coefficient matrix A, the variable matrix X, and the constant matrix B. Then

$$AX = B$$

Multiply both sides by the inverse of A to obtain the solution.

$$(A^{-1})AX = (A^{-1})B$$
$$[(A^{-1})A]X = (A^{-1})B$$
$$IX = (A^{-1})B$$
$$X = (A^{-1})B$$

Question: If the coefficient matrix does not have an inverse, does that mean the system has no solution?
Answer: No, if the coefficient matrix is not invertible, the system could be inconsistent and have no solution, or be dependent and have infinitely many solutions.

Example: Solving a 2 × 2 System Using the Inverse of a Matrix
Solve the given system of equations using the inverse of a matrix.

$$3x + 8y = 5$$
$$4x + 11y = 7$$

Solution: Write the system in terms of a coefficient matrix, a variable matrix, and a constant matrix.

$$A = \begin{bmatrix} 3 & 8 \\ 4 & 11 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

Then

$$\begin{bmatrix} 3 & 8 \\ 4 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

First, we need to calculate A^{-1}. Using the formula to calculate the inverse of a 2 by 2 matrix, we have:

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$= \frac{1}{3(11) - 8(4)}\begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix}$$
$$= \frac{1}{1}\begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix}$$

So,

$$A^{-1} = \begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix}$$

Now we are ready to solve. Multiply both sides of the equation by A^{-1}.

$$(A^{-1})AX = (A^{-1})B$$

$$\begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 3 & 8 \\ 4 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11(5) + (-8)7 \\ -4(5) + 3(7) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The solution is $(-1, 1)$.

Exercises for Friday, April 17, 2020
9.7 Exercises page 841 #27

ANSWER KEY

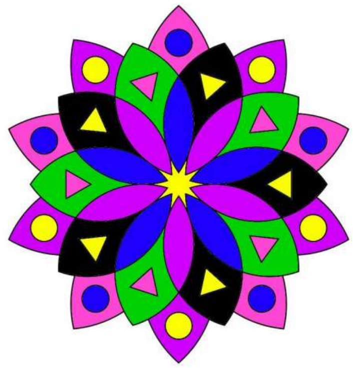
Exercises for Monday, April 13, 2020:
HOLIDAY

Exercises for Tuesday April 14, 2020

Basic Matrix Operations Color by Number—Answer Key

Directions: Simplify each matrix below. Then, color the given specific entry on the picture with the indicated color

Matrix	Solution	Entry
1. $\begin{bmatrix} 1 & 0 \\ 2 & -3 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -5 \\ -1 & 1 \\ -4 & -5 \end{bmatrix}$	$\begin{bmatrix} 3 & \boxed{5} \\ 3 & -4 \\ 3 & 2 \end{bmatrix}$	a_{12} pink
2. $-2\begin{bmatrix} 5 & 15 & -2 \\ 3 & -2 & -1 \end{bmatrix}$	$\begin{bmatrix} -10 & \boxed{-30} & 4 \\ -6 & 4 & 2 \end{bmatrix}$	a_{12} purple
3. $[5 \quad 2] + [-3 \quad -7]$	$[\boxed{2} \quad -5]$	a_{11} green
4. $-5[3 \quad -4 \quad 0 \quad 2]$	$[-15 \quad 20 \quad 0 \quad \boxed{-10}]$	a_{14} blue
5. $-2x[4x \quad y^2 \quad -7x \quad 5]$	$[-8x^2 \quad -2xy^2 \quad 14x^2 \quad \boxed{-10x}]$	a_{14} yellow
6. $10\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$	$\begin{bmatrix} 20 \\ 30 \\ \boxed{-20} \end{bmatrix}$	a_{31} black
7. $[2 \quad -7 \quad -6] + [1 \quad -8 \quad -1]$	$[\boxed{3} \quad -15 \quad -7]$	a_{11} pink
8. $3\begin{bmatrix} 3x \\ 2y^2 \\ x \end{bmatrix}$	$\begin{bmatrix} 9x \\ 6y^2 \\ \boxed{3x} \end{bmatrix}$	a_{31} purple
9. $\begin{bmatrix} -6x+y \\ -x \\ 6z \end{bmatrix} + \begin{bmatrix} 6x \\ 3y \\ 2-x \end{bmatrix}$	$\begin{bmatrix} \boxed{y} \\ -x+3y \\ 6z-x+2 \end{bmatrix}$	a_{11} green
10. $5[3 \quad 2 \quad 2 \quad -7] - [2 \quad 4 \quad -6 \quad 2]$	$[13 \quad 6 \quad \boxed{16} \quad -37]$	a_{13} blue
11. $\begin{bmatrix} 4 & 3 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} -6 & 7 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} -3 & 4 \\ 1 & -6 \end{bmatrix}$	$\begin{bmatrix} 13 & -8 \\ 2 & \boxed{11} \end{bmatrix}$	a_{22} yellow
12. $\begin{bmatrix} 2 & 3 \\ -4 & -4 \\ 6 & 8 \end{bmatrix} + 5\begin{bmatrix} -9 & 3 \\ 6 & 1 \\ -4 & -3 \end{bmatrix}$	$\begin{bmatrix} -43 & 18 \\ 26 & \boxed{1} \\ -14 & -7 \end{bmatrix}$	a_{22} black



Exercises for Wednesday April 15, 2020

$$\textbf{1. } AB = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1(-3) + 4(1) & 1(-4) + 4(1) \\ -1(-3) + -3(1) & -1(-4) + -3(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} -3(1) + -4(-1) & -3(4) + -4(-3) \\ 1(1) + 1(-1) & 1(4) + 1(-3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Exercises for Thursday, April 16, 2020

$$\textbf{13. } \frac{1}{29}\begin{bmatrix} 9 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\textbf{15. } \frac{1}{69}\begin{bmatrix} -2 & 7 \\ 9 & 3 \end{bmatrix}$$

$$\textbf{21. } \frac{1}{17}\begin{bmatrix} -5 & 5 & -3 \\ 20 & -3 & 12 \\ 1 & -1 & 4 \end{bmatrix}$$

Exercises for Friday, April 17, 2020

27. (-5,6)

Name:

Pre-Calculus

QUIZ

Show your work on each problem.

1. Determine whether (15, 10) is a solution to the following system:

$$6x - 3y = 20$$
$$-2x + 3y = 20$$

2. For the following, perform the matrix addition.

$$\begin{bmatrix} 1 & 5 \\ 8 & 92 \\ 12 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 14 \\ 7 & 2 \\ 5 & 61 \end{bmatrix}$$

3. Rewrite the system of linear equations as an augmented matrix.

$$14x - 2y + 13z = 140$$
$$-2x + 3y - 6z = -1$$
$$x - 5y + 12z = 11$$

4. Rewrite the augmented matrix as a system of linear equations.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 12 \\ -2 & 4 & 9 & -5 \\ -6 & 1 & 2 & 8 \end{array}\right]$$