Pre-Calculus: Week of April 6 - 9

Time Allotment: 40 minutes per day

Student Name: ________________________________

Teacher Name: Mrs. Melisa R. Walters

Zoom office hours:

3rd Period	4th Period	6th Period
1-1:50 pm on Mondays and Wednesdays	10-10:50 am on Tuesdays and Thursdays	1-1:50 pm on Tuesdays and Thursdays

Packet Overview

Date	Objective(s)	Page Number
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Additional Notes: We miss you all very much!

I hope that you are all staying safe and keeping that positive attitude that you always bring with you to class. Any questions? Email me directly: melisa.walters@greatheartsnorthernoaks.org

Each lesson will end with a set of math problems pertaining to that particular lesson of the day.

Please create an "Exercise Packet" which is to include all your work and completion of these daily exercises. Each day is to have a title with the date followed by the name of the lesson. Please include a title page and staple all the completed exercises. At a later point, we will ask you to turn your exercise packet. Do not worry right now about whether that will be online or in person, simply do the problem set as I instruct with the proper titles and labels.

Thank you for your hard work students. I appreciate all of you. Have a great day!

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

____

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

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Monday, April 6

Pre-Calculus Unit: Chapter 9
Lesson 1: Matrices and Matrix Operations

Objective: 1. Write the augmented Matrix of a System of Linear Equations,
2. Express the system from the Matrix

NOTE: I will be providing some examples specific to this lesson. Please highlight the key points as you read through this entire lesson or take notes in the margin. At the end of the lesson, the list of questions in the exercise are to be completed to ensure mastery.

Preparing for this section:
Consider the following system of linear equations:

$$x + 4y = 14$$
$$3x - 2y = 0$$

If we choose **not** to write the symbols used for the variables, we can represent this system as:

$$\left[\begin{array}{cc|c} 1 & 4 & 14 \\ 3 & -2 & 0 \end{array}\right]$$

where it is understood that the first column represents the coefficients of the variable x, the second column the coefficients of y, and the third column the constants on the right side of the equal signs. The vertical line serves as a reminder of the equal signs. The large square brackets are used to denote a *matrix* in algebra.
If a matrix *A* has *m* rows and *n* columns, then it is written as $A = [\ a_{ij}\ ]_{m\times n}\ , 1 \le i \le m, 1 \le j \le n$.

That is,

$$A=[a_{ij}]_{m\times n}=\begin{array}{c} \overset{n\text{ columns}}{\begin{array}{cccccc}\text{Column 1}&\text{Column 2}&&\text{Column } j&&\text{Column } n\end{array}} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}\end{array}\begin{array}{l}\leftarrow \text{Row 1}\\ \leftarrow \text{Row 2}\\ \\ \leftarrow \text{Row } i\\ \\ \leftarrow \text{Row } m\end{array} \quad m\text{ rows}$$

Each number $[\ a_{ij}\ ]$ of the matrix has two indexes; the **row index** *i* and the **column index** *j*
The matrix shown in display above has *m* rows and *n* columns. The numbers a_{ij} are usually referred to as the **entries** of the matrix. For example, a_{23} refers to the entry in the second row, third column.

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Question: Finding the Dimensions of the Given Matrix and Locating Entries

Given matrix A:

a. What are the dimensions of matrix A?

b. What are the entries at a_{31} and a_{22} ?

Answer:

a. The dimensions are 3 × 3 because there are three rows and three columns.
b. b. Entry a_{31} is the number at row 3, column 1, which is 3.
   The entry a_{22} is the number at row 2, column 2, which is 4.

*******Remember, the row comes first, then the column.

Let's write the augmented matrix of a system of linear equations together.

Now, we will use matrix notation to represent a system of linear equations. The matrices (plural of matrix) used to represent systems of linear equations are called **augmented matrices**. In writing the augmented matrix of a system, the variables of each equation must be on the left side of the equal sign and the constants on the right side. A variable that does not appear in an equation has a coefficient of 0.

Example: Write the augmented matrix of each system of equations.

$$3x - 4y = -6$$
$$2x - 3y = -5$$

Solution: The augmented matrix is

$$\begin{bmatrix} 3 & -4 & | & -6 \\ 2 & -3 & | & -5 \end{bmatrix}$$

Example: Write the augmented matrix of each system of equations.

$$2x - y + z = 0$$
$$x + z - 1 = 0$$
$$x + 2y - 8 = 0$$

First, you must write all the constants and place them on the other side of the equal sign.
Then, you write a 0 for any variable that does not appear in the equation.

$$2x - y + z = 0$$
$$x + 0y + z = 1$$
$$x + 2y + 0z = 8$$

Solution: The augmented matrix is $\begin{bmatrix} 2 & -1 & 1 & 0 \\ 1 & 0 & 1 \,|\, & 1 \\ 1 & 2 & 0 & 8 \end{bmatrix}$

Now that you know how to write a matrix from a set of equations. Let's learn how to write a set of equations when given a matrix.

Steps: Given a system of equations, write an augmented matrix.

1. Write the coefficients of the x-terms as the numbers down the first column.
2. Write the coefficients of the y-terms as the numbers down the second column.
3. If there are z-terms, write the coefficients as the numbers down the third column.
4. Draw a vertical line and write the constants to the right of the line.

Example: Write the system of linear equations corresponding to each augmented matrix.

$$\left[\begin{array}{cc|c} 5 & 2 & 13 \\ -3 & 1 & -10 \end{array}\right]$$

Solution: The matrix has two rows and so represents a system of two equations. The two columns to the left of the vertical bar indicate that the system has two variables. If x and y are the two variables used to denote the variables, the system of equations is

$$5x + 2y = 13$$
$$-3x + y = -10$$

Perform Row Operations (Adding and Subtracting) on a Matrix

<u>Adding and Subtracting Matrices</u>

We use matrices to list data or to represent systems. Because the entries are numbers, we can perform operations on matrices. We add or subtract matrices by adding or subtracting corresponding entries. In order to do this, the entries must correspond. Therefore, addition and subtraction of matrices is **only possible when the matrices have the same dimensions**. We can add or subtract a 3 × 3 matrix and another 3 × 3 matrix, but we cannot add or subtract a 2 × 3 matrix and a 3 × 3 matrix because some entries in one matrix will not have a corresponding entry in the other matrix.

adding and subtracting matrices

Given matrices A and B of like dimensions, addition and subtraction of A and B will produce matrix C or matrix D of the same dimension.

$$A + B = C \text{ such that } a_{ij} + b_{ij} = c_{ij}$$
$$A - B = D \text{ such that } a_{ij} - b_{ij} = d_{ij}$$

Matrix addition is commutative.

$$A + B = B + A$$

It is also associative.

$$(A + B) + C = A + (B + C)$$

Example: Finding the Sum of Matrices

Find the sum of A and B, given

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

Solution: Add corresponding entries

$$A + B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$$= \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Example: Adding Matrix A and Matrix B
Find the sum of A and B.

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 9 \\ 0 & 7 \end{bmatrix}$$

Solution: Add corresponding entries. Add the entry in row 1, column 1, a_{11}, of matrix A to the entry in row 1, column 1, b_{11}, of B. Continue the pattern until all entries have been added.

$$A + B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 9 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4+5 & 1+9 \\ 3+0 & 2+7 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 10 \\ 3 & 9 \end{bmatrix}$$

Example: Finding the Difference of Two Matrices

Find the difference of A and B.

$$A = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 & 1 \\ 5 & 4 \end{bmatrix}$$

Solution: We subtract the corresponding entries of each matrix.

$$A - B = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 1 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 - 8 & 3 - 1 \\ 0 - 5 & 1 - 4 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 2 \\ -5 & -3 \end{bmatrix}$$

Example: Finding the Sum and Difference of Two 3 x 3 Matrices

Given A and B :

a. Find the sum.

b. Find the difference.

$$A = \begin{bmatrix} 2 & -10 & -2 \\ 14 & 12 & 10 \\ 4 & -2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 10 & -2 \\ 0 & -12 & -4 \\ -5 & 2 & -2 \end{bmatrix}$$

Solution:

a. Add the corresponding entries.

$$A + B = \begin{bmatrix} 2 & -10 & -2 \\ 14 & 12 & 10 \\ 4 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 10 & -2 \\ 0 & -12 & -4 \\ -5 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 6 & -10 + 10 & -2 - 2 \\ 14 + 0 & 12 - 12 & 10 - 4 \\ 4 - 5 & -2 + 2 & 2 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & -4 \\ 14 & 0 & 6 \\ -1 & 0 & 0 \end{bmatrix}$$

b. Subtract the corresponding entries.

$$A - B = \begin{bmatrix} 2 & -10 & -2 \\ 14 & 12 & 10 \\ 4 & -2 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 10 & -2 \\ 0 & -12 & -4 \\ -5 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 6 & -10 - 10 & -2 + 2 \\ 14 - 0 & 12 + 12 & 10 + 4 \\ 4 + 5 & -2 - 2 & 2 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -20 & 0 \\ 14 & 24 & 14 \\ 9 & -4 & 4 \end{bmatrix}$$

Please click on this video for a more in-depth explanation:
https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:matrices/x9e81a4f98389efdf:adding-and-subtracting-matrices/v/matrix-addition-and-subtraction-1

Directions for completing the exercises:

1. Create a packet for completed exercises. Please staple together several papers with a title page "Exercises for Pre-Calculus"
2. At the top of each exercise completion page, title as such: Exercises for Monday, April 6, 2020 Matrices and Matrix Operations.
3. After you finish the entire exercise, please check your answers with the answer key at the end of the packet and attempt correction.

Exercises for Monday, April 6, 2020:

For the following exercises, use the matrices below and perform the matrix addition or subtraction. Indicate if the operation is undefined.

$$A=\begin{bmatrix}1 & 3\\0 & 7\end{bmatrix}, B=\begin{bmatrix}2 & 14\\22 & 6\end{bmatrix}, C=\begin{bmatrix}1 & 5\\8 & 92\\12 & 6\end{bmatrix}, D=\begin{bmatrix}10 & 14\\7 & 2\\5 & 61\end{bmatrix}, E=\begin{bmatrix}6 & 12\\14 & 5\end{bmatrix}, F=\begin{bmatrix}0 & 9\\78 & 17\\15 & 4\end{bmatrix}$$

1. C + D
2. B – E
3. D – B

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Tuesday, April 7

Pre-Calculus: chapter 9
Lesson 2: Finding Scalar Multiples of a matrix

Objective: 1. Perform Row operations (products) on a matrix,
2. Solve a system of linear equations using Matrices

NOTE: I will be providing some examples specific to this lesson. Please highlight the key points as you read through this entire lesson or take notes in the margin. At the end of the lesson, the list of questions in the exercise are to be completed to ensure mastery.

Lesson 2

Besides adding and subtracting whole matrices, there are many situations in which we need to multiply a matrix by a constant called a **scalar**. Recall that a scalar is a real number quantity that has magnitude, but not direction. For example, time, temperature, and distance are scalar quantities. The process of scalar multiplication involves multiplying each entry in a matrix by a scalar. A **scalar multiple** is any entry of a matrix that results from scalar multiplication.

scalar multiplication

Scalar multiplication involves finding the product of a constant by each entry in the matrix. Given

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

the scalar multiple cA is

$$\begin{aligned} cA &= c\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ &= \begin{bmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{bmatrix} \end{aligned}$$

Scalar multiplication is distributive. For the matrices A, B, and C with scalars a and b,

$$a(A + B) = aA + aB$$

$$(a + b)A = aA + bA$$

Example: Multiplying the Matrix by a Scalar

Multiply matrix A by the scalar 3

$$A = \begin{bmatrix} 8 & 1 \\ 5 & 4 \end{bmatrix}$$

Solution: Multiply each entry in A by the scalar 3

$$\begin{aligned} 3A &= 3\begin{bmatrix} 8 & 1 \\ 5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 \cdot 8 & 3 \cdot 1 \\ 3 \cdot 5 & 3 \cdot 4 \end{bmatrix} \\ &= \begin{bmatrix} 24 & 3 \\ 15 & 12 \end{bmatrix} \end{aligned}$$

Your turn to try it:

Given matrix B, find −2B where

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

Please check your answer in the answer key before continuing the lesson.
Example: **Finding the Sum of Scalar Multiples**

Find the sum 3A + 2B.

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 2 \\ 4 & 3 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 0 & -3 & 2 \\ 0 & 1 & -4 \end{bmatrix}$$

Solution:

First, find $3A$, then $2B$.

$$\begin{aligned} 3A &= \begin{bmatrix} 3\cdot 1 & 3(-2) & 3\cdot 0 \\ 3\cdot 0 & 3(-1) & 3\cdot 2 \\ 3\cdot 4 & 3\cdot 3 & 3(-6) \end{bmatrix} \\ &= \begin{bmatrix} 3 & -6 & 0 \\ 0 & -3 & 6 \\ 12 & 9 & -18 \end{bmatrix} \\ 2B &= \begin{bmatrix} 2(-1) & 2\cdot 2 & 2\cdot 1 \\ 2\cdot 0 & 2(-3) & 2\cdot 2 \\ 2\cdot 0 & 2\cdot 1 & 2(-4) \end{bmatrix} \\ &= \begin{bmatrix} -2 & 4 & 2 \\ 0 & -6 & 4 \\ 0 & 2 & -8 \end{bmatrix} \end{aligned}$$

Now, add 3A + 2B.

$$3A + 2B = \begin{bmatrix} 3 & -6 & 0 \\ 0 & -3 & 6 \\ 12 & 9 & -18 \end{bmatrix} + \begin{bmatrix} -2 & 4 & 2 \\ 0 & -6 & 4 \\ 0 & 2 & -8 \end{bmatrix}$$
$$= \begin{bmatrix} 3-2 & -6+4 & 0+2 \\ 0+0 & -3-6 & 6+4 \\ 12+0 & 9+2 & -18-8 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -2 & 2 \\ 0 & -9 & 10 \\ 12 & 11 & -26 \end{bmatrix}$$

Example: Finding the Product of Two Matrices

In addition to multiplying a matrix by a scalar, we can multiply two matrices. Finding the product of two matrices is only possible when the inner dimensions are the same, meaning that the number of columns of the first matrix is equal to the number of rows of the second matrix. If A is an m × r matrix and B is an r × n matrix, then the product matrix AB is an m × n matrix. For example, the product AB is possible because the number of columns in A is the same as the number of rows in B. If the inner dimensions do not match, the product is not defined.

$$\underset{2 \times 3}{A} \cdot \underset{3 \times 3}{B}$$
$$\text{same}$$

We multiply entries of A with entries of B according to a specific pattern as outlined below. The process of matrix multiplication becomes clearer when working a problem with real numbers. To obtain the entries in row i of AB, we multiply the entries in row i of A by column j in B and add. For example, given matrices A and B, where the dimensions of A are 2 × 3 and the dimensions of B are 3 × 3, the product of AB will be a 2 × 3 matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Multiply and add as follows to obtain the first entry of the product matrix AB.

1. . To obtain the entry in row 1, column 1 of AB, multiply the first row in A by the first column in B, and add.

$$[a_{11} \quad a_{12} \quad a_{13}]\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}$$

2. To obtain the entry in row 1, column 2 of AB, multiply the first row of A by the second column in B, and add.

$$[a_{11} \quad a_{12} \quad a_{13}]\begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} = a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + a_{13} \cdot b_{32}$$

3. To obtain the entry in row 1, column 3 of AB, multiply the first row of A by the third column in B, and add.

$$[a_{11} \quad a_{12} \quad a_{13}]\begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = a_{11} \cdot b_{13} + a_{12} \cdot b_{23} + a_{13} \cdot b_{33}$$

We proceed the same way to obtain the second row of AB. In other words, row 2 of A times column 1 of B; row 2 of A times column 2 of B; row 2 of A times column 3 of B. When complete, the product matrix will be

$$AB = \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + a_{13} \cdot b_{32} & a_{11} \cdot b_{13} + a_{12} \cdot b_{23} + a_{13} \cdot b_{33} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} + a_{23} \cdot b_{32} & a_{21} \cdot b_{13} + a_{22} \cdot b_{23} + a_{23} \cdot b_{33} \end{bmatrix}$$

properties of matrix multiplication

For the matrices A, B, and C the following properties hold.

- Matrix multiplication is associative: $(AB)C = A(BC)$.
- Matrix multiplication is distributive: $C(A + B) = CA + CB,$ $(A + B)C = AC + BC.$

Note that matrix multiplication is not commutative.

Example: Multiplying Two Matrices

Multiply matrix A and matrix B.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Solution: First, we check the dimensions of the matrices. Matrix A has dimensions 2 × 2 and matrix B has dimensions 2 × 2. The inner dimensions are the same so we can perform the multiplication. The product will have the dimensions 2 × 2. We perform the operations outlined previously.

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \\ &= \begin{bmatrix} 1(5)+2(7) & 1(6)+2(8) \\ 3(5)+4(7) & 3(6)+4(8) \end{bmatrix} \\ &= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \end{aligned}$$

Example: Multiplying Two Matrices

Given A and B:

a. Find AB.
b. Find BA.

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{bmatrix}$$

Solution:

a. As the dimensions of A are 2 × 3 and the dimensions of B are 3 × 2, these matrices can be multiplied together because the number of columns in A matches the number of rows in B. The resulting product will be a 2 × 2 matrix, the number of rows in A by the number of columns in B.

$$\begin{aligned} AB &= \begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -1(5)+2(-4)+3(2) & -1(-1)+2(0)+3(3) \\ 4(5)+0(-4)+5(2) & 4(-1)+0(0)+5(3) \end{bmatrix} \\ &= \begin{bmatrix} -7 & 10 \\ 30 & 11 \end{bmatrix} \end{aligned}$$

b. The dimensions of B are 3 × 2 and the dimensions of A are 2 × 3. The inner dimensions match so the product is defined and will be a 3 × 3 matrix.

$$\begin{aligned} BA &= \begin{bmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 5(-1)+-1(4) & 5(2)+-1(0) & 5(3)+-1(5) \\ -4(-1)+0(4) & -4(2)+0(0) & -4(3)+0(5) \\ 2(-1)+3(4) & 2(2)+3(0) & 2(3)+3(5) \end{bmatrix} \\ &= \begin{bmatrix} -9 & 10 & 10 \\ 4 & -8 & -12 \\ 10 & 4 & 21 \end{bmatrix} \end{aligned}$$

Analysis Notice that the products AB and BA are not equal.

$$AB = \begin{bmatrix} -7 & 10 \\ 30 & 11 \end{bmatrix} \neq \begin{bmatrix} -9 & 10 & 10 \\ 4 & -8 & -12 \\ 10 & 4 & 21 \end{bmatrix} = BA$$

This illustrates the fact that matrix multiplication is not commutative.
Question: Is it possible for AB to be defined but not BA?

Answer: Yes, consider a matrix A with dimension 3 × 4 and matrix B with dimension 4 × 2. For the product AB the inner dimensions are 4 and the product is defined, but for the product BA the inner dimensions are 2 and 3 so the product is undefined.

Steps to using a calculator: Given a matrix operation, evaluate using a calculator. 1.

1. Save each matrix as a matrix variable [A], [B], [C], ...
2. Enter the operation into the calculator, calling up each matrix variable as needed.
3. If the operation is defined, the calculator will present the solution matrix; if the operation is undefined, it will display an error message.

Example: Using a Calculator to Perform Matrix Operations
Find $AB - C$ given

$$A = \begin{bmatrix} -15 & 25 & 32 \\ 41 & -7 & -28 \\ 10 & 34 & -2 \end{bmatrix}, B = \begin{bmatrix} 45 & 21 & -37 \\ -24 & 52 & 19 \\ 6 & -48 & -31 \end{bmatrix}, \text{and } C = \begin{bmatrix} -100 & -89 & -98 \\ 25 & -56 & 74 \\ -67 & 42 & -75 \end{bmatrix}.$$

Solution: On the matrix page of the calculator, we enter matrix A above as the matrix variable [A], matrix B above as the matrix variable [B], and matrix C above as the matrix variable [C]. On the home screen of the calculator, we type in the problem and call up each matrix variable as needed.

[A]×[B] − [C]

The calculator gives us the following matrix.

$$\begin{bmatrix} -983 & -462 & 136 \\ 1{,}820 & 1{,}897 & -856 \\ -311 & 2{,}032 & 413 \end{bmatrix}$$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Please click on this video for a more in-depth explanation:
https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:matrices/x9e81a4f98389efdf:multiplying-matrices-by-scalars/v/scalar-multiplication

Exercises for Tuesday, April 7, 2020

1. For the following exercises, use the matrices below to perform scalar multiplication.
   3B.

$$B = \begin{bmatrix} 3 & 9 \\ 21 & 12 \\ 0 & 64 \end{bmatrix}$$

For the following exercises, use the matrices below to perform matrix multiplication.

$$A = \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 6 & 4 \\ -8 & 0 & 12 \end{bmatrix}, C = \begin{bmatrix} 4 & 10 \\ -2 & 6 \\ 5 & 9 \end{bmatrix}, D = \begin{bmatrix} 2 & -3 & 12 \\ 9 & 3 & 1 \\ 0 & 8 & -10 \end{bmatrix}$$

2. BC
3. BD

<u>**Wednesday, April 8, 2020**</u>

Pre-Calculus: chapter 9

Lesson 3: Solving Systems with Gaussian Elimination

Objective: Convert the matrix to row-echelon form

NOTE: I will be providing some examples specific to this lesson. Please highlight the key points as you read through this entire lesson or take notes in the margin. At the end of the lesson, the list of questions in the exercise are to be completed to ensure mastery.

Lesson 3:
Now that we can write systems of equations in augmented matrix form, we will examine the various **row operations** that can be performed on a matrix, such as addition, multiplication by a constant, and interchanging

rows. Performing row operations on a matrix is the method we use for solving a system of equations. In order to solve the system of equations, we want to convert the matrix to **row-echelon form**, in which there are ones down the **main diagonal** from the upper left corner to the lower right corner, and zeros in every position below the main diagonal as shown.

Row-echelon form

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & d \\ 0 & 0 & 1 \end{bmatrix}$$

We use row operations corresponding to equation operations to obtain a new matrix that is row-equivalent in a simpler form. Here are the guidelines to obtaining row-echelon form.

1. In any nonzero row, the first nonzero number is a 1. It is called a leading 1.
2. Any all-zero rows are placed at the bottom on the matrix.
3. Any leading 1 is below and to the right of a previous leading 1.
4. Any column containing a leading 1 has zeros in all other positions in the column

To solve a system of equations we can perform the following row operations to convert the coefficient matrix to row-echelon form and do back-substitution to find the solution.

1. Interchange rows. (Notation: Ri ↔ Rj)
2. Multiply a row by a constant. (Notation: cRi)
3. Add the product of a row multiplied by a constant to another row. (Notation: Ri + cRj)

Each of the row operations corresponds to the operations we have already learned to solve systems of equations in three variables. With these operations, there are some key moves that will quickly achieve the goal of writing a matrix in row-echelon form. To obtain a matrix in row-echelon form for finding solutions, we use Gaussian elimination, a method that uses row operations to obtain a 1 as the first entry so that row 1 can be used to convert the remaining rows.

Gaussian elimination

The **Gaussian elimination** method refers to a strategy used to obtain the row-echelon form of a matrix. The goal is to write matrix A with the number 1 as the entry down the main diagonal and have all zeros below.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow{\text{After Gaussian elimination}} A = \begin{bmatrix} 1 & b_{12} & b_{13} \\ 0 & 1 & b_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

The first step of the Gaussian strategy includes obtaining a 1 as the first entry, so that row 1 may be used to alter the rows below.

Given an augmented matrix, perform row operations to achieve row-echelon form.

1. The first equation should have a leading coefficient of 1. Interchange rows or multiply by a constant, if necessary.
2. Use row operations to obtain zeros down the first column below the first entry of 1.
3. Use row operations to obtain a 1 in row 2, column 2.
4. Use row operations to obtain zeros down column 2, below the entry of 1.
5. Use row operations to obtain a 1 in row 3, column 3.
6. Continue this process for all rows until there is a 1 in every entry down the main diagonal and there are only zeros below.
7. If any rows contain all zeros, place them at the bottom.

Example: Solving a 2 × 2 System by Gaussian Elimination

Solve the given system by Gaussian elimination.

$$2x + 3y = 6$$

$$x - y = \frac{1}{2}$$

Solution: First, we write this as an augmented matrix.

$$\left[\begin{array}{cc|c} 2 & 3 & 6 \\ 1 & -1 & \frac{1}{2} \end{array}\right]$$

We want a 1 in row 1, column 1. This can be accomplished by interchanging row 1 and row 2.

$$R_1 \leftrightarrow R_2 \rightarrow \left[\begin{array}{cc|c} 1 & -1 & \frac{1}{2} \\ 2 & 3 & 6 \end{array}\right]$$

We now have a 1 as the first entry in row 1, column 1. Now let's obtain a 0 in row 2, column 1. This can be accomplished by multiplying row 1 by −2, and then adding the result to row 2.

$$-2R_1 + R_2 = R_2 \rightarrow \left[\begin{array}{cc|c} 1 & -1 & \frac{1}{2} \\ 0 & 5 & 5 \end{array}\right]$$

We only have one more step, to multiply row 2 by 1/5

$$\frac{1}{5}R_2 = R_2 \rightarrow \left[\begin{array}{cc|c} 1 & -1 & \frac{1}{2} \\ 0 & 1 & 1 \end{array}\right]$$

Use back-substitution. The second row of the matrix represents y = 1. Back-substitute y = 1 into the first equation.

$$x - (1) = \frac{1}{2}$$

$$x = \frac{3}{2}$$

The solution is the point

$$\left(\frac{3}{2}, 1\right).$$

Example: Using Gaussian Elimination to Solve a System of Equations

Use Gaussian elimination to solve the given 2 × 2 system of equations.

$$2x + y = 1$$
$$4x + 2y = 6$$

Solution: Write the system as an augmented matrix

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 4 & 2 & 6 \end{array}\right]$$

Obtain a 1 in row 1, column 1. This can be accomplished by multiplying the first row by 1/ 2 .

$$\frac{1}{2}R_1 = R_1 \rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 4 & 2 & 6 \end{array}\right]$$

Next, we want a 0 in row 2, column 1. Multiply row 1 by −4 and add row 1 to row 2.

$$-4R_1 + R_2 = R_2 \rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 4 \end{array}\right]$$

The second row represents the equation 0 = 4. Therefore, the system is inconsistent and has no solution.

Please click on this video for a more in-depth explanation:
https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:matrices/x9e81a4f98389efdf:row-echelon-and-gaussian-elimination/v/matrices-reduced-row-echelon-form-2

Exercise for Wednesday April 8, 2020

Solve the given system by Gaussian elimination.

$$4x + 3y = 11$$
$$x - 3y = -1$$

Thursday, April 9

Pre-Calculus: chapter 9
Lesson 4: More practice problems on Gaussian Elimination method

Objective: Matrix method for solving a system of equations of linear equations in Row Echelon Form.

NOTE: I will be providing some examples specific to this lesson. Please highlight the key points as you read through this entire lesson or take notes in the margin. At the end of the lesson, the list of questions in the exercise are to be completed to ensure mastery.

Lesson 4
Example: Solving a Dependent System
Solve the system of equations.

$$3x + 4y = 12$$
$$6x + 8y = 24$$

Solution: Perform row operations on the augmented matrix to try and achieve row-echelon form

$$A = \left[\begin{array}{cc|c} 3 & 4 & 12 \\ 6 & 8 & 24 \end{array}\right]$$

$$-\frac{1}{2}R_2 + R_1 = R_1 \rightarrow \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 6 & 8 & 24 \end{array}\right]$$

$$R_1 \leftrightarrow R_2 \rightarrow \left[\begin{array}{cc|c} 6 & 8 & 24 \\ 0 & 0 & 0 \end{array}\right]$$

The matrix ends up with all zeros in the last row: 0y = 0. Thus, there are an infinite number of solutions and the system is classified as dependent. To find the generic solution, return to one of the original equations and solve for y.

$$3x + 4y = 12$$
$$4y = 12 - 3x$$
$$y = 3 - \frac{3}{4}x$$

So the solution to this system is $\left(x, 3 - \frac{3}{4}x\right)$.

Example: Performing Row Operations on a 3 × 3 Augmented Matrix to Obtain Row-Echelon Form
Perform row operations on the given matrix to obtain row-echelon form.

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 2 & -5 & 6 & 6 \\ -3 & 3 & 4 & 6 \end{array}\right]$$

Solution: The first row already has a 1 in row 1, column 1. The next step is to multiply row 1 by −2 and add it to row 2. Then replace row 2 with the result

$$-2R_1 + R_2 = R_2 \rightarrow \left[\begin{array}{rrr|r} 1 & -3 & 4 & 3 \\ 0 & 1 & -2 & 0 \\ -3 & 3 & 4 & 6 \end{array}\right]$$

Next, obtain a zero in row 3, column 1.

$$3R_1 + R_3 = R_3 \rightarrow \left[\begin{array}{rrr|r} 1 & -3 & 4 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & -6 & 16 & 15 \end{array}\right]$$

Next, obtain a zero in row 3, column 2.

$$6R_2 + R_3 = R_3 \rightarrow \left[\begin{array}{rrr|r} 1 & -3 & 4 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 4 & 15 \end{array}\right]$$

The last step is to obtain a 1 in row 3, column 3.

$$\frac{1}{2}R_3 = R_3 \rightarrow \left[\begin{array}{rrr|r} 1 & -3 & 4 & 3 \\ 0 & 1 & -2 & -6 \\ 0 & 0 & 1 & \frac{21}{2} \end{array}\right]$$

Question: Can any system of linear equations be solved by Gaussian elimination?
Answer: Yes, a system of linear equations of any size can be solved by Gaussian elimination.

Exercise for Thursday, April 2, 2020

Write the system of equations in row-echelon form.

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Friday, April 10 - Holiday

ANSWER KEY

Exercises for Monday, April 6, 2020:

1.

$$\begin{bmatrix} 11 & 19 \\ 15 & 94 \\ 17 & 67 \end{bmatrix}$$

2.

$$\begin{bmatrix} -4 & 2 \\ 8 & 1 \end{bmatrix}$$

3. Undefined; dimensions do not match

Try it Exercises for Tuesday April 7, 2020

Answer:

$$-2B = \begin{bmatrix} -8 & -2 \\ -6 & -4 \end{bmatrix}$$

Exercises for Tuesday April 7, 2020

Answers:

1.

$$\begin{bmatrix} 9 & 27 \\ 63 & 36 \\ 0 & 192 \end{bmatrix}$$

2.

$$\begin{bmatrix} 20 & 102 \\ 28 & 28 \end{bmatrix}$$

3.

$$\begin{bmatrix} 60 & 41 & 2 \\ -16 & 120 & -216 \end{bmatrix}$$

Exercises for Wednesday April 8, 2020

Answer: (2, 1)

Exercises for Thursday, April 9, 2020

Answer:

$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{2} & \frac{5}{2} & \frac{17}{2} \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right]$$