

**Euclidean Geometry**

April 27 – May 1

*Time Allotment: 40 minutes per day*

Student Name: \_\_\_\_\_

Teacher Name: \_\_\_\_\_

**Packet Overview**

Date	Objective(s)	Page Number
Monday, April 27	1. Ratios and proportions explained	2
Tuesday, April 28	1. Ratios and proportions explained further	4
Wednesday, April 29	1. Means & Extremes	7
Thursday, April 30	1. Definitions	9
Friday, May 1	1. Minor Assessment	11

**Additional Notes:** Hello Students!

This week we will be going back to Euclid’s Elements.

Make sure you are reading carefully as you go through these lessons with a pencil in your hand (NO PENS). You should always be underlining, circling, taking margin notes etc.

**Do all of your work on sheet of notebook paper. You can keep your packet, but you will need to turn in/scan the work you do on a piece of notebook paper.**

Mr. Bernstein will have office hours at the following times

- 1<sup>st</sup> Period 10:00-10:50am Mondays & Wednesdays
- 5<sup>th</sup> Period 11:00- 11:50 am Tuesdays & Thursdays

Miss McCafferty will hold office hours at the following times:

- 1<sup>st</sup> Period 10:00-10:50 am Mondays & Wednesdays
- 3<sup>rd</sup> Period 1:00- 1:50 pm Mondays & Wednesdays
- 4<sup>th</sup> Period 10:00-10:50 am Tuesdays & Thursdays
- 6<sup>th</sup> Period 1:00- 1:50 pm Tuesdays & Thursdays

**Love,**

Miss McCafferty and Mr. Bernstein

**The answer key to each lesson will be at the end of each lesson. The answer keys should only be used when checking work.**

**Academic Honesty**

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

*Student signature:*

\_\_\_\_\_

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

*Parent signature:*

\_\_\_\_\_

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

**Monday, April 27**

Geometry Unit: Ratio & Proportion

Lesson 1: Ratio and Proportion

**Objective:** Be able to do this by the end of this lesson.

1. Set up a proportion using triangles

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

**Notes**

Read and annotate the following passage. You will answer reading comprehension questions about the passage after you read and annotate.

**Background on ratio and proportion**

Book V covers the abstract theory of **ratio** and **proportion**. A ratio is an indication of the *relative size of two magnitudes*. A magnitude is anything that has *size*. The propositions in the following book, Book VI, are all geometric and depend on ratios, so the theory of ratios needs to be developed first. To get a better understanding of what ratios are in geometry, consider the first proposition VI.1 (pg. 123 Euclid's Elements). It states that triangles of the same height are proportional to their bases, that is to say, one triangle's size, or area, is to another as the length of one base is to the other. (A proportion is simply an equality of two ratios.)

(This reading will be continued on March 28<sup>th</sup>.)

Examples of Bolded Terms

Ratio Examples:

- A : B
- 1 : 2
- $\pi$  : radius,
- hypotenuse : side of the square

Proportion Examples:

- 1:2 :: 2:4
- $\frac{1}{2} = \frac{4}{8}$
- 2 Puppies : 1 dog ::  
2 Kittens : 1 cat

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**EXERCISE 1:** (do your drawings, calculations, and proportion work on a separate sheet of paper titled: April 27, Exercise 1)

1. Draw a right isosceles triangle. Draw the two equal sides so that they are 1 inch long each. Name the triangle ABC
2. Draw a right isosceles triangle. Draw the two equal sides so that they are 2 inches long each. Name the triangle QRS
3. Calculate the area of ABC & QRS
4. Write a proportion with the following information:

**(Base of ABC) : (Base of QRS) :: (Area of ABC) : (Area of QRS)**

The way you *read* proportions out loud would be to say “The base of ABC is to the Base of QRS as the Area of ABC is to the Area of QRS”

*Watch the video on google classroom! I will explain this further.*

Another way of looking at talking about/looking at ratios is to put them in fraction form:

$$\frac{\text{Base of } ABC}{\text{Base of } QRS} = \frac{\text{Area of } ABC}{\text{Area of } QRS}$$

All numerical proportions can be translated to fractions. In Euclid, however, we are going to do our best to get comfortable working with proportions that are written like this:

$$\mathbf{A : B :: C : D}$$

**Exercise 2: Make flashcards for definitions 1- 8 on page 99**

**Tuesday, April 28**

Geometry Unit: Ratio & Proportion

Lesson 2: Ratio and Proportion

**Objective:** Be able to do this by the end of this lesson.

1. Explain what means and extremes are

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

**Notes**

*You will need to use your notes and work from yesterday as a reference for today's reading.*

March 27<sup>th</sup> Reading continued:

Look at your triangles from March 27<sup>th</sup>. A simple example is when one base is twice the other, therefore the triangle on that base is also twice the triangle on the other base. This ratio of 2:1 is fairly easy to comprehend. Indeed, any ratio equal to a ratio of two numbers is easy to comprehend. Given a proportion that says a ratio of lines equals a ratio of numbers, for instance,

$$A : B = 8:5$$

Let's stop and look at this proportion real quick

$$A : B = 8:5$$

Another way we can write this is

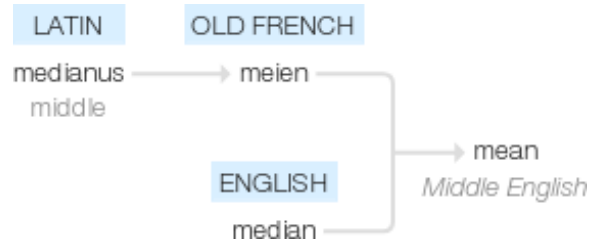
$$A : B :: 8:5$$

In all proportions the first term and the last term (they are written in light blue above) are called the **EXTREMES**. They are called the **EXTREMES** because they are on the outside of the proportion. They are physically on the extremities of the proportion.

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

$$A : B :: 8:5$$

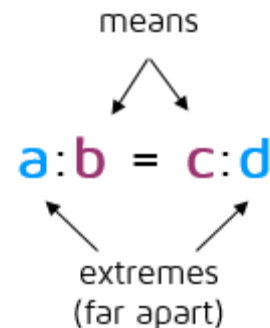
In all proportions the second and third term (they are written in black above) are called the **MEANS**. This comes from the Latin word meadianus, which is translated as middle. (see the graphic to the right of this paragraph)



In Pre- Algebra you learned that in proportions the means multiplied equals the extreme multiplied.

### Exercise 1

1. Look at the proportion to the right. Use this proportion to complete the following exercise:



Write an equation that sets the extremes multiplied equal to

the means multiplied by filling in the information in the example below.

$$extreme \times extreme = mean \times mean$$

Do the same thing you did for number 1 with the following proportions

2.  $1 : 2 :: 3 : 6$
3.  $4 : 5 :: 8 : 10$

Now Cross multiply the following equations and write out your answer as an equation.

4.  $\frac{1}{2} = \frac{3}{6}$
5.  $\frac{4}{5} = \frac{8}{10}$

6. Look closely at what you just did for # 2-5. In 4-5 sentences explain how # 2-3 are similar to # 4- 5.

## Euclidean Geometry

April 27 – May 1

Answer key to Lesson 2

2.  $a \times d = b \times c$

3.  $1 \times 6 = 2 \times 3$

$$6 = 6$$

4.  $4 \times 10 = 8 \times 5$

$$40 = 40$$

5.  $1 \times 6 = 2 \times 3$

$$6 = 6$$

6.  $4 \times 10 = 8 \times 5$

$$40 = 40$$

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

**Wednesday, April 29**

Geometry Unit: Ratio & Proportion

Lesson 3: Ratio and Proportion

**Objective:** Be able to do this by the end of this lesson.

1. Explain what means and extremes are

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

**Notes**

March 28<sup>th</sup> reading continued:

Let's look back at the proportion from the past couple of days:

$$A : B :: 8:5$$

If we turn it into a fraction it will look like this

$$\frac{A}{B} = \frac{8}{5}$$

If we multiply the means and the extremes of the proportion above we can also look at it as:

$$5 \times A = 8 \times B$$

If we cross multiply the fractions above **WE GET THE SAME THING**

I like to think of fractions as ratios that have been rotated. Proportions and fractional equations are very similar. Do not let proportions scare you. You have been working with proportions your whole life you just did not know it. Book V is going to give you the vocabulary to talk about something you have dealt with your whole life.

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.



Now, let's look back at  $A : B :: 8:5$  if  $A : B$  equals a ratio of numbers then  $A$  and  $B$  are commensurable, that is, both are measured by a common measure. Many straight lines, however, are not commensurable. Some straight lines will never go evenly into another straight line. (example: the hypotenuse and the side of the square) If  $A$  is the side of a square and  $B$  its diagonal, then  $A$  and  $B$  are not commensurable; the ratio  $A : B$  is not the ratio of numbers. This fact seems to have been discovered by the Pythagoreans, perhaps Hippasus of Metapontum, some time before 400 B.C.E., a hundred years before Euclid's Elements.

The difficulty of commensurable magnitudes is one of foundations of Book V: what is an adequate definition of proportion that includes the incommensurable case? The solution is that in V.Def.5. That definition, and the whole theory of ratio and proportion in Book V, are attributed to Eudoxus of Cnidus (died. ca. 355 B.C.E.) We will talk more about Definition 5 later!

**Use definitions 1- 8 on page 99 of the Elements to answer the following questions:**

1. Based on your readings this week and the definitions, how would you define magnitude?
2. A \_\_\_\_\_ is a sort of relation in respect to size between two magnitudes of the same kind.
3. A magnitude is a \_\_\_\_\_ of a magnitude, the less of the greater, when it measures the greater.
4. The greater is a \_\_\_\_\_ of the less when it is measured by the less.
5. Magnitudes that are said to be in the same ratio are called \_\_\_\_\_.
6. A proportion in \_\_\_\_\_ terms is the least possible.

**Thursday, April 30**

Geometry Unit: Ratio & Proportion

Lesson 4: Ratio and Proportion

**Objective:** Be able to do this by the end of this lesson.

1. Explain Euclid’s terms and definitions

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

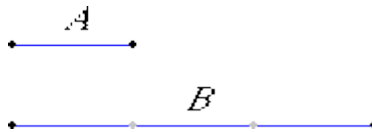
**Guided Walk-Through Definitions**

**Definition 1**

A magnitude is a part of a magnitude, the less of the greater, when it measures the greater.

**Definition 2**

The greater is a multiple of the less when it is measured by the less.



**Guide**

The two magnitudes mentioned in each definition are same kind of magnitude.

Following Euclid, they are illustrated here as lines, but they could both be planar figures, or solids, or angles, or any other kind of magnitude so long as they are of the **same kind**.

The illustration shows two magnitudes, A and B, and A is one third of B since A measures B three times. Thus, A is a part of B, and B is a multiple of A.

**Definition 3**

A ratio is a sort of relation in respect of size between two magnitudes of the same kind.

**Guide**

A convenient notation for a ratio of two magnitudes A and B of the same kind is A : B.

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

In Euclid there are **No mixed ratios**. All ratios will be between two magnitudes of the same kind. For example a line : a line. A triangle : a triangle. You will never see a ratio between a line and a triangle. It does not happen in Euclid. This is similar to how we talked about equality at the beginning of the year. You cannot say a triangle = a line. A line and a triangle are not the same kind of thing.

All of Euclid's ratios are pure ratios of two magnitudes of the same kind, **there are no mixed ratios** in the Elements. A familiar example of a mixed ratio is velocity, the ratio of a distance to a time, measured in units such as kilometers/hour.

That isn't to say that ratios of different kinds of magnitudes aren't equated. In fact, that's one of the more important aspects of ratios. For example, the fundamental proposition of Book VI, proposition VI.1, says that given two triangles of the same height, the ratio of the triangles  $A : B$  is the same as the ratio of their heights  $Ah : Bh$ . That says that the ratio of two plane figures equals the ratio of two lines. Proportions can be mixed **RATIOS CANNOT BE MIXED**.

### Operations on ratios and proportions, compounded ratios

There are several operations on ratios and proportions defined soon. For instance, Book V.Def.9 defines duplicate ratios, under certain circumstances, which may be thought of as the squares of ratios. See also definitions V.Def.12 through V.18. But ratios are neither numbers nor magnitudes, and the usual operations of addition, subtraction, multiplication, and division that apply to numbers don't apply to ratios.

Numbers can be added and subtracted, and so can magnitudes of the same kind, but ratios cannot. Take for example a ratio  $A : B$  of plane figures and a ratio  $C : D$  of angles. What could be meant by their sum  $(A : B) + (C : D)$ ? NOTHING it is nonsense. It does not work.

One obvious approach is to treat ratios as quotients. That suggests  $A/B + C/D = (AD + BC)/BD$ , but a product of a plane figure and an angle, such as  $AD$ , has no meaning, so the obvious approach has obvious difficulties.

Don't worry if this all seems confusing it will make more sense as we go along I promise!

### Assignment: Study for quiz tomorrow!

You have a quiz on Friday on definitions 1 – 8. You should spend 5-10 mins each day going through your flash cards for definitions 1-8. You will not be allowed to use notes on your quiz.

**Friday, May 1**

Geometry Unit: Ratio & Proportion

Lesson 5: Minor Assessment

**Objective:** Be able to do this by the end of this lesson.

1. Complete Minor assessment

Do not use any notes. Do this from memory.

Geometry  
Book V: Definitions 1-8

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. A magnitude is a part of a magnitude, the less of the greater, when it \_\_\_\_\_ the greater.
2. The great is a \_\_\_\_\_ of the less when it is measured by the less.
3. A \_\_\_\_\_ is a sort of relation in respect of \_\_\_\_\_ between two magnitudes of the same kind.
4. Let magnitudes which have the same ratio be called \_\_\_\_\_.
5. Write Euclid's definition of a proportion.