GreatHearts

Euclidean Geometry

May 4 – 8

Time Allotment: 40 minutes per day

Student Name: ________________________________

Teacher Name: ________________________________

Euclidean Geometry
May 4 – May 8

Packet Overview

Date	Objective(s)	Page Number
Monday, May 4	1. Ratios and proportions explained	2
Tuesday, May 5	1. Ratios and proportions explained further	4
Wednesday, May 6	1. Means & Extremes	7
Thursday, May 7	1. Definitions	9
Friday, May 8	1. Minor Assessment	11

Additional Notes: Hello students!,

This week we will be going back to Euclid's Elements.

Make sure you are reading carefully as you go through these lessons with a pencil in your hand (NO PENS). You should always be underlining, circling, taking margin notes etc.

Do all of your work on sheet of notebook paper. You can keep your packet, but you will need to turn in/scan the work you do on a piece of notebook paper.
Mr. Bernstein will have Guided Instruction at the following times

- 1st Period 10:00-10:50am Mondays & Wednesdays
- 5th Period 11:00- 11:50 am Tuesdays & Thursdays

Miss McCafferty will hold Guided Instruction at the following times:

- 1st Period 10:00-10:50 am Mondays & Wednesdays
- 3rd Period 1:00- 1:50 pm Mondays & Wednesdays
- 4th Period 10:00-10:50 am Tuesdays & Thursdays
- 6th Period 1:00- 1:50 pm Tuesdays & Thursdays

Love,
Miss McCafferty and Mr. Bernstein
The answer key to each lesson will be at the end of each lesson. The answer keys should only be used when checking work.

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

____

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

____

Monday, May 4

Geometry Unit: Ratio & Proportion
Lesson 1: Ratio and Proportion

Objective: Be able to do this by the end of this lesson.

1. Justify why Horn angles and Rectilinear angles aren't capable of being in a Euclidean Ratio

Definition 4

Magnitudes are said to have a ratio to one another which can, when multiplied, exceed one another.

Guide

This definition limits the existence of ratios to comparable magnitudes of the same kind where comparable means each, when multiplied, can exceed the other. The ratio doesn't exist when one magnitude is so small or the other so large that no multiple of the one can exceed the other. This definition excludes the ratio of a finite straight line to an infinite straight line and the ratio of an infinitesimal straight line, should any exist, to a finite straight line.

The result on horn angles in proposition III.16 excludes ratios between horn angles and rectilinear angles. That proposition states that a horn angle is less than any rectilinear angle, hence no multiple of a horn angle is greater than a rectilinear angle. The situation of horn angles is much worse than that, however, since horn angles of different sizes aren't even comparable.

Ratios must be between magnitudes which are the same *kind*. If they are the same kind of thing and one magnitude can be multiplied and exceed the other magnitude, then they can be in a ratio together.

Exercise 1: Can the following magnitudes be in a ratio together? (Answer yes or no)

1. A line and a Triangle
2. A line and a line
3. A triangle and a Cube
4. A number and a line

Definition 4 as an axiom of comparability

This definition is used repeatedly as an axiom for magnitudes rather than a definition. It is frequently invoked in this book, starting with proposition V.8 but also required for more fundamental properties, and elsewhere, such as the important proposition X.1. In the proofs of these propositions one magnitude is less than another, and it is asserted that some multiple of the smaller is greater than the larger. Euclid implicitly assumes that the magnitudes he discusses, except horn angles, are all comparable. Straight lines, rectilinear angles, plane figures, and solids are all comparable to any other of the same **type.**

This principle of comparability should be explicit in order to justify the principle of comparability for magnitudes of these kinds. One solution is to make it a postulate that straight lines are comparable. From that postulate comparability of each of the other kinds of magnitudes could be proved.

Several of the propositions, stated and unstated, depend on this principle. Without it, some are simply false for kinds of magnitude that have infinitesimals. If x and y are two magnitudes of the same kind, then x is *infinitesimal* with respect to y, or y is *infinite* with respect to x, if no multiple of x is greater than y.

Exercise 2

1. Draw a horn angle & a rectilinear angle
2. Why can't horn angles and rectilinear angles be in a ratio together? (Answer in 3-5 complete sentences)

Answer Key

Page 2

1. No
2. Yes
3. No
4. No

Tuesday, May 5

Geometry Unit: Ratio & Proportion

Lesson 2: Ratio and Proportion

Objective: Be able to do this by the end of this lesson.

1. Identify terms in a proportion
2. Take Equimultiples

Definition 5

Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever are taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

We are going to break this down piece by piece

Terms:

Equimultiple, this just means that **EQUAL** multiples have been taken. If I have 4 cookies and you have two and I multiply my cookies by 4 and you multiply yours by 4, then we have **EQUALLY** multiplied our cookies. We don't have equal amounts of cookies but we have multiplied our cookies by the same number.

In proportions Euclid will refer to the 1st, 2nd, 3rd, etc. terms in proportions. All you need to do is write out the proportion and look at the order that the terms are in.

If the proportion is Q : R :: X : Z, then the first term is Q, the second is R, etc.

Exercise 1

1. What is the first term in the proportion A : B :: D : C?
2. What is the third term?
3. What are the means of the proportion?
4. What are the extremes?
5. What would the proportion look like if we swapped the 1st and 4th terms?

6. A : D :: B : Q

    a. Re-Write the proportion for number 6 where you multiply the 1st and 3rd terms by the same number

Answer Key pg. 4

1. A
2. D
3. B & D
4. A & C
5. C : B :: D : A

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Wednesday, May 6

Geometry Unit: Ratio & Proportion
Lesson 3: Ratio and Proportion

Objective: Be able to do this by the end of this lesson.

1. Determine whether or not ratios are proportional

Definition 5

Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever are taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

Guide

We will use the example of w : x :: y : z for definition 5

Now lets multiply the first and third terms (w & y) by n, and multiply the second and fourth term (x & z) by m.

Euclid's definition is giving us a way to test and see it these ratios are in the same ratio. If they first and third term always behave the same way, with respect to the second and fourth term, then the ratios are proportional!

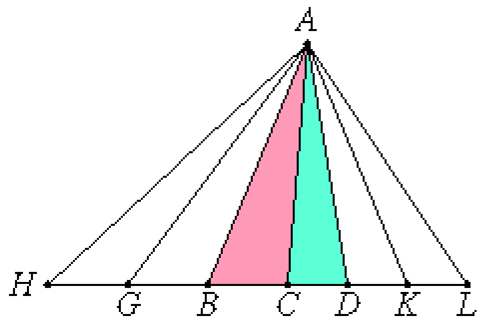
if $nw > mx$, then $ny > mz$,
if $nw = mx$, then $ny = mz$, and
if $nw < mx$, then $ny < mz$.

It is very convenient to use the shorter notation

if nw >=< mx, then ny >=< mz.

Note that whenever the symbol >=< is used there are three parallel statements being made.

The four magnitudes, in a proportion, do not all have to be of the same kind, but the first pair *w* and *x* need to be of the one kind, and the second pair *y* and *z* of one kind, either the same kind as that of *w* and *x* or a different kind. Perhaps the best illustration of these definitions comes from proposition VI.1 in which Euclid first uses them to construct a proportion.



The goal in this proposition is to show that the lines are proportional to the triangles. More precisely, the line *BC* is to the line *CD* as the triangle *ABC* is to the triangle *ACD*, that is, the ratio *BC* : *CD* of lines is the same as the ratio *ABC* : *ACD* of triangles. Even though the ratios derive from different kinds of magnitudes, they are to be compared and shown equal.

According to Definition 5, in order to show the ratios are the same, Euclid takes any one multiple of *BC* and *ABC* (which he illustrates by taking three times each), and any one multiple of *CD* and *ACD* (which he also illustrates by taking three times each). Then he proceeds to show that the former equimultiples, namely *HC* and *CL*, alike exceed, are alike equal to, or alike fall short of, the latter equimultiples, namely, *AHC* and *ACL*.

Symbolically, in order to prove *BC* : *CD* = *ABC* : *ACD*, Euclid proves for any numbers *n* and *m* that the line *n BC* is greater, equal, or less than the line *m CD* when the triangle *n ABC* is greater, equal, or less than the triangle *m ACD.* We will abbreviate this condition symbolically as

if *n BC* >=< *m CD,* then *n ABC* >=< *m ACD.*

Note that in order to check this condition, it is only necessary to compare lines to lines and planar figures to planar figures. To see how Euclid does this, refer to VI.1.

Short Answer

1. I have been very clear that ratios must be between objects of the same kind. Is it possible for two lines to be in the same ratio as two cubes? Why or why not? (Answer in 5-6 complete sentences.)

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Thursday, May 7

Geometry Unit: Ratio & Proportion
Lesson 4: Ratio and Proportion

Objective: Be able to do this by the end of this lesson.

1. Assess which ratio is greatest

Definition 7

When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to have a greater ratio to the second than the third has to the fourth.

Guide

Definition 5 explained when two ratios were equal, namely, $w : x = y : z$ when for all numbers n and m, if $nw >=< mx$, then $ny >=< mz$.

Definition 7 now says $w : x > y : z$ when

there are numbers n and m such that $nw > mx$ but ny is not greater than mz.

Of course, $y : z$ is called the lesser ratio.

Example:

Lets look at the two ratios 2:3 and 1:4

w = 2 y = 1 n = 5

x = 3 z = 4 m = 2

Now we want to take equimultiples of the first and third terms and different equimultiples of the 2nd and 4th terms.

$$5 \times 2 : 2 \times 3 \qquad 5 \times 1 : 2 \times 4$$

$$10 : 6 \qquad 5 : 8$$

Now we see here that 10 > 6

But 5 is not greater than 8

Therefore 10 : 6 in a greater ration than 5: 8

And therefore 2 : 3 > 1: 4

Another shortcut to figure out which is the greater ratio is to translate them into fractions and see which fraction is larger.

Exercise 1

Which of these ratios is greater?

1. 5 : 3   2 : 3
2. 1 : 2   4 : 5
3. 6 : 3   5 : 2

Answer Key pg. 9

1. 5 : 3
2. 4 : 5
3. 6 : 3

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Friday, May 8

Geometry Unit: Ratio & Proportion
Lesson 5: Minor Assessment

Objective: Take minor assessment

1. If *A:B::C:D* then what would this ratio look like if you swapped the 2nd and 4th terms?

 ________:_________::__________:__________

2. If *E:F::G:H* then what are the means of the proportion?

3. A magnitude is a part of a magnitude, the less of the greater, when it______________________ the greater.

4. The great is a ________________________ of the less when it is measured by the less.

5. A ______________________ is a sort of relation in respect of _______________ between two magnitudes of the same kind.

6. Let magnitudes which have the same ratio be called ____________________.