Calculus I

May 18 – May 22

Time Allotment: 40 minutes per day

Student Name: ______________________________

Teacher Name: ______________________________

Packet Overview

Date	Objective(s)	Page Number
Monday, May 18	Review: Finding Area Between Curves	2-6
Tuesday, May 19	Review: 3D Rotation Using Disc Method	7-9
Wednesday, May 20	Review: 3D Rotation Using Washer Method	10-11
Thursday, May 21	Review: Euler's Number	12-15
Friday, May 22	Minor Assessment	16-18

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

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I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

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Monday, May 18

Calculus Unit: Review: Finding Area Between Curves Practice

Objective: Be able to do this by the end of this lesson.
1. Practice Solving Problems Asking to Find Area Between Two Curves.
2. Find intersection points to determine bounds of definite integral.
3. Sketch graphs of curves to verify you are subtracting the correct function from the other.
4. Determine whether the area you have found is reasonable given the graph.

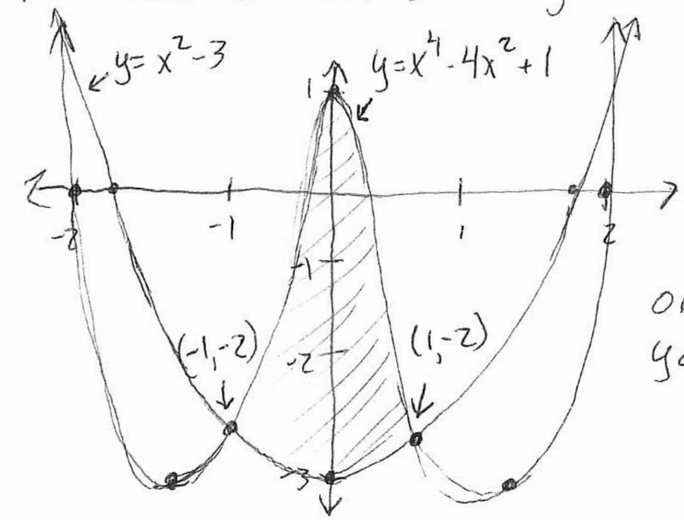
Introduction to Lesson 1

Today we're going to review finding the area between functions. You'll be given a graph for some exercises, and some you will just be given the two functions. In the latter case, always sketch a graph of the functions before you start finding intersection points or set up the integral.

Lesson 2 - 1

Let's do an example problem and then let you try some practice exercises on your own!

Ex:



Here's a sketch of two functions,

$y = x^4 - 4x + 1$

$y = x^2 - 3$

I would recommend graphing them on your calculator or Desmos so you can see it better.

Our goal is to find the area of the shaded region.

First, let's find our lower and upper bounds (or you can call them the left and right-most boundaries). These will be the intersection points. If you plug in -1 to both functions, you will get -2. If you plug in 1 to both functions, you will get -2, so these are our intersection points. -1 is our lower bound, and 1 is our upper bound. $\int_{-1}^{1} (\quad) \, dx$

Second, we need to fill in the parentheses. To do this, we'll take the upper function, $y = x^4 - 4x + 1$ and subtract the lower function, $y = x^2 - 3$.

$$\int_{-1}^{1} \left(x^4 - 4x^2 + 1 - (x^2 - 3)\right) dx$$

Try working this integral on a separate sheet of scratch paper. Then check your answer.

$$= \int_{-1}^{1} (x^4 - 5x^2 + 4) \, dx$$

$$= \frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x \Big|_{-1}^{1}$$

$$= \frac{1}{5} - \frac{5}{3} + 4 - \left(-\frac{1}{5} + \frac{5}{3} - 4\right)$$

$$= \frac{2}{5} - \frac{10}{3} + 8 = 8 + \frac{6}{15} - \frac{50}{15}$$

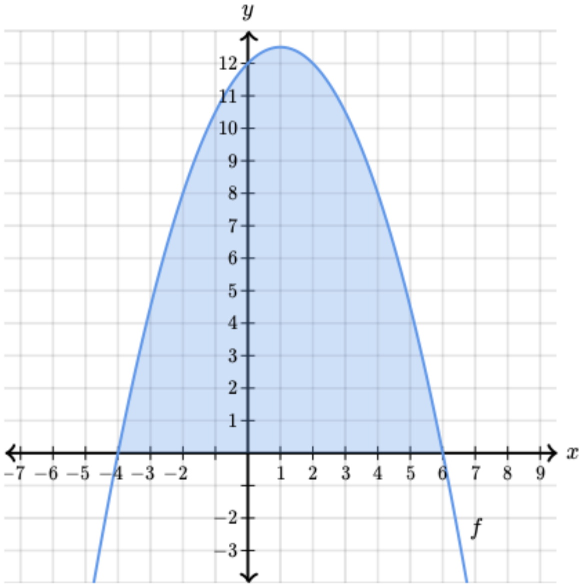
$$\boxed{= 8 - \frac{44}{15} = \frac{76}{15}}$$

Calculus I – Lesson 2 Exercises : Finding Areas Under, Above, and Between Curves

1)

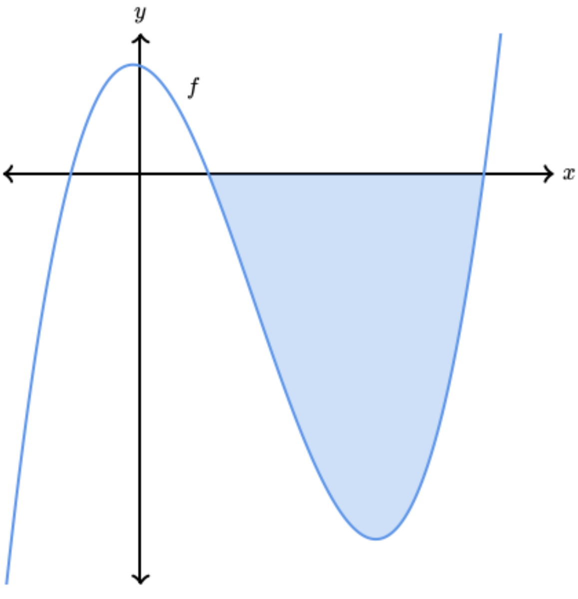
$f(x) = 12 + x - \frac{1}{2}x^2$

and the x-axis.



What is its area?

2) The shaded region is bounded by the graph of the function $f(x) = x^3 - 5x^2 - x + 5$ and the x-axis.

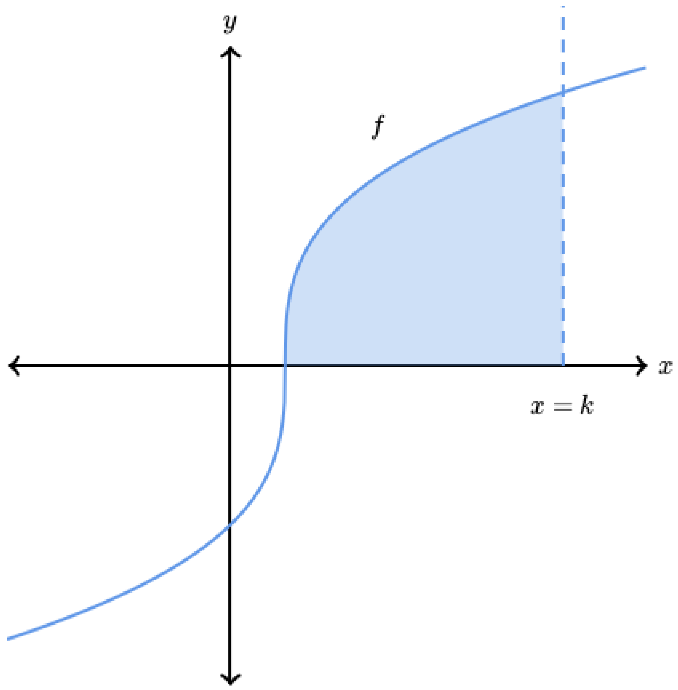


What are the zeroes of this function?

Find the shaded area if the zeros of the function are -1, 1, and 5.

3)

The shaded region is bounded by the graph of the function $f(x) = \sqrt[3]{x-1}$, the line $x = k$, and the x-axis.



If the region has area 12, what is the *exact* value of k?

4)

What is the area of the region between the graphs of $f(x) = 2x^2 + 5x$ and $g(x) = -x^2 - 6x + 4$ from $x = -4$ to $x = 0$?

5)

What is the area of the region between the graphs of $f(x) = \sqrt{x + 10}$ and $g(x) = x - 2$ from $x = -10$ to $x = 6$?

6)

What is the area of the region between the graphs of $f(x) = x^2 - 3x$ and $g(x) = 2x$ from $x = 0$ to $x = 5$?

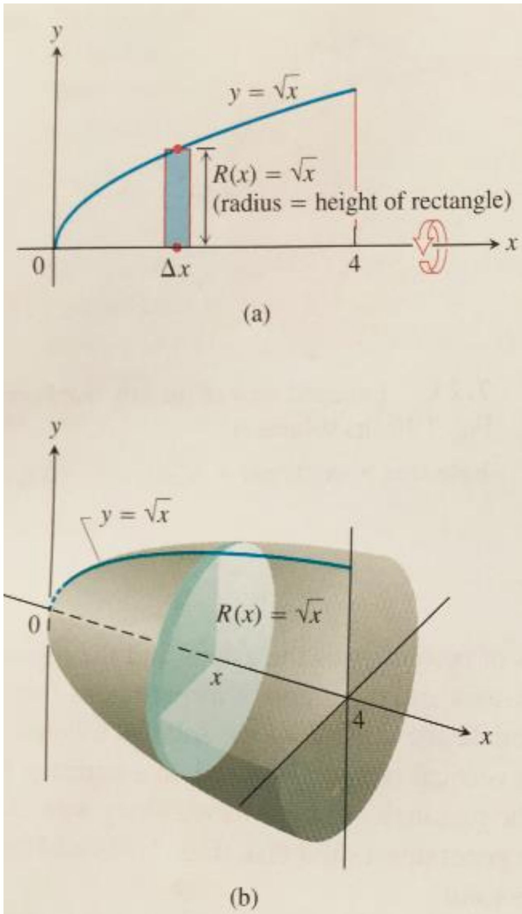
Tuesday, May 19

Calculus Unit: Applications of Integrals
Lesson 2: Rotating using the disc method.

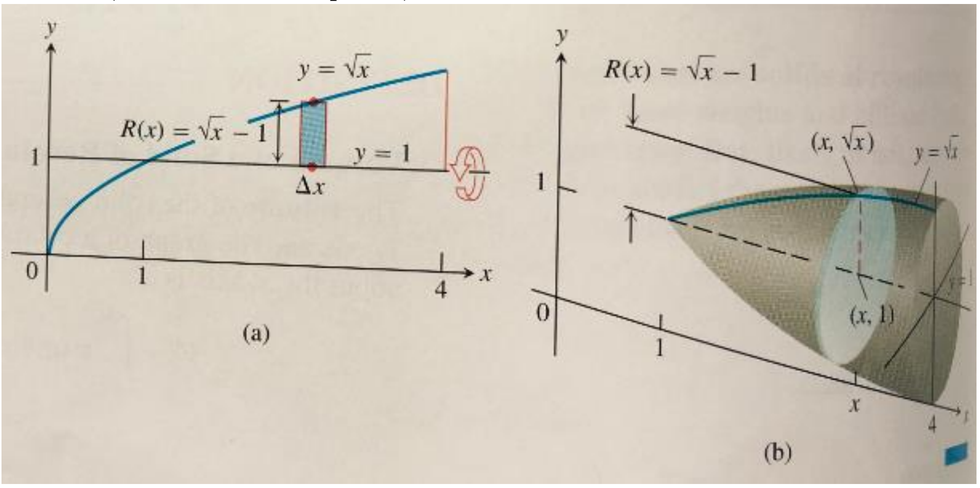
Objective: Be able to do this by the end of this lesson.
1. Find volume using disc method while accounting for the axis of rotation (be it the x-axis, y-axis, or a different axis of rotation..

Introduction to Lesson 2
Today we'll review using the disc method for rotating objects about different axes.

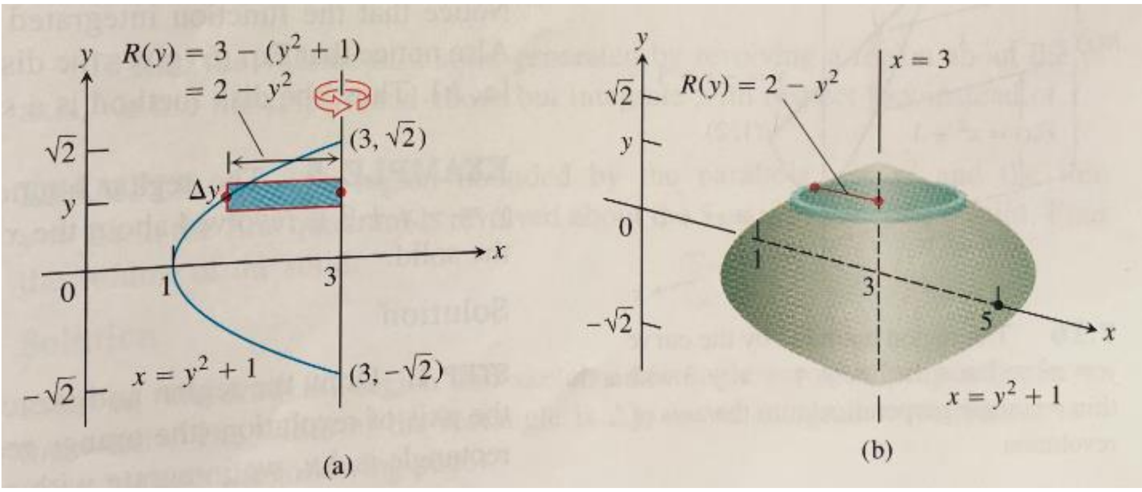
1. Let's take the curve $y = \sqrt{x}$ between $0 \leq x \leq 4$, and revolve the function about the x-axis to generate the solid below. Find the volume of that solid (and then check your answer on p. 448).



2. Now let's find the volume when the same function is rotated about the y = 1 axis, with the same interval. (check solution on p. 448)



3. And now one where you have to use dy thicknesses and rotate about the vertical line x = 3 (check answer on p. 449).



On a separate sheet of paper, do numbers 1, 3, and 9 on p. 453 (Section 7-2).

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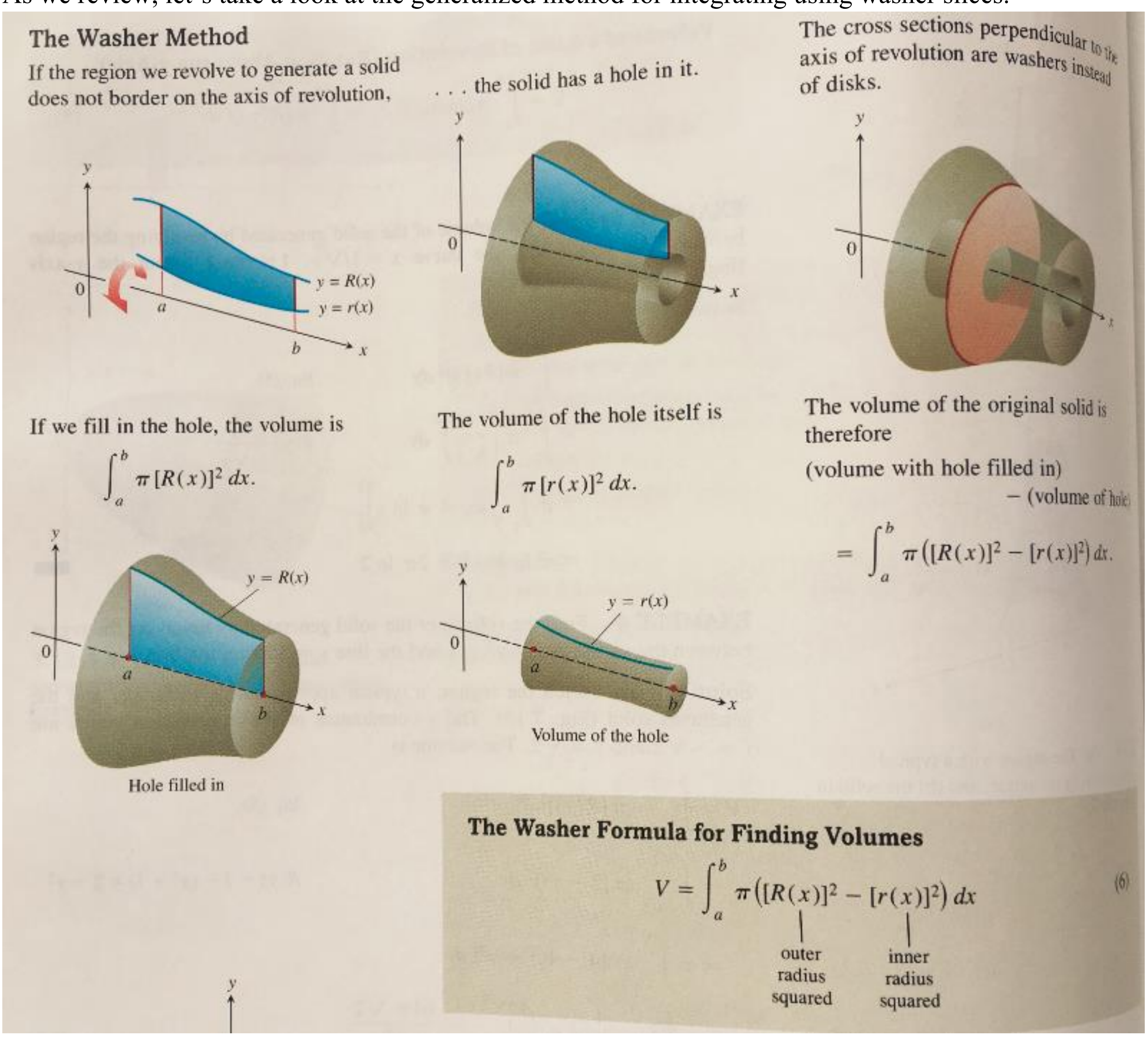
Wednesday May 20

Calculus Unit: Applications of Integrals
Lesson 3: Finding Volume of Using the Washer Method

Objective: Be able to do this by the end of this lesson.
1. Review using the washer method when integrating a rotation about the x-axis or y-axis.
2. Review using the washer method when integrating a rotation about an arbitrary vertical or horizontal line.

Introduction to Lesson 3
Today we're going to review the washer method. As you remember, the key is finding the two radii of the washer and then subtracting the bigger circle area from the smaller circle area.

As we review, let's take a look at the generalized method for integrating using washer slices.


The Washer Method

If the region we revolve to generate a solid does not border on the axis of revolution, . . . the solid has a hole in it. The cross sections perpendicular to the axis of revolution are washers instead of disks.

If we fill in the hole, the volume is

$$\int_a^b \pi [R(x)]^2 \, dx.$$

The volume of the hole itself is

$$\int_a^b \pi [r(x)]^2 \, dx.$$

The volume of the original solid is therefore

(volume with hole filled in) − (volume of hole)

$$= \int_a^b \pi \left([R(x)]^2 - [r(x)]^2\right) dx.$$

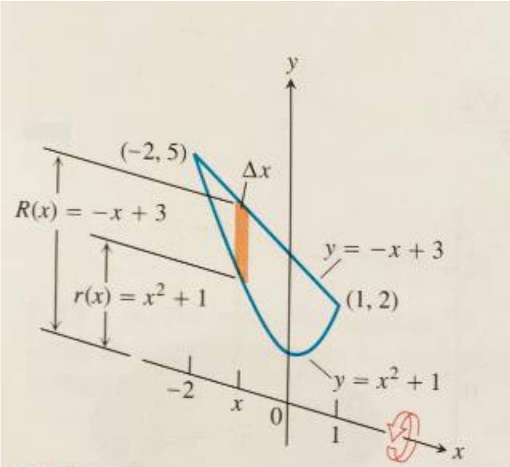
Hole filled in

Volume of the hole

The Washer Formula for Finding Volumes

$$V = \int_a^b \pi \left(\underbrace{[R(x)]^2}_{\text{outer radius squared}} - \underbrace{[r(x)]^2}_{\text{inner radius squared}}\right) dx \qquad (6)$$

1. Now let's work a washer problem: The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the out and inner radii, then the surface area of the washer, then volume of the washer, and finally the volume of the solid. (check your answer on p. 451).



7.16 The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ with a thin rectangle perpendicular to the axis of revolution.

2. This is a washer problem that requires you to integrate with respect to y and use dy slices. Try to draw the graph on your own. If you need help, you can follow the steps on p. 452.

The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

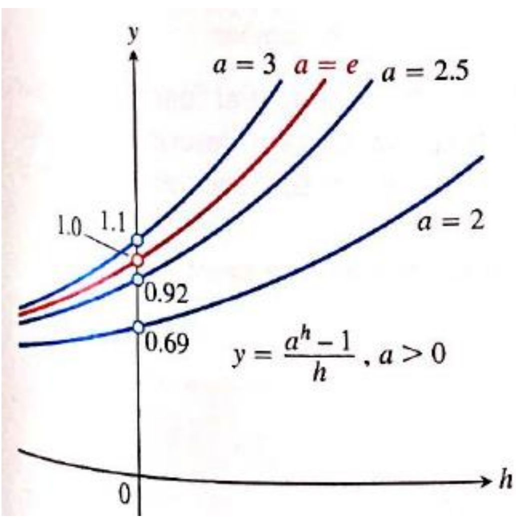
On a separate sheet of paper, do exercises 25, 27, and 37 on p. 453 (Section 7-2).

Thursday May 21

Calculus Unit: Transcendental Functions
Lesson 3: The Discovery of e.

Objective: Be able to do this by the end of this lesson.
1. Find digits of a base of an exponential function whose derivative is equal to itself.
2. Find the slope of the function $y = e^x$ at various points.

Introduction to Lesson 4
Remember how we explored applying the definition of the derivative to take the derivative of an exponential function, $y = a^x$, where a is a positive number. It turned out that the derivative of any function of that form is equal to La^x, where L is some constant. But then we wondered if there was a base number, a, such that when we took the derivative, L would equal 1. We saw that when we took base numbers that were in between 2.5 and 3 and took their derivatives, L got closer to 1, as seen in the graph below.



$$\frac{d}{dx}(2^x) \approx (0.69)\,2^x,$$
$$\frac{d}{dx}\left(2.5^x\right) \approx (0.92)\,2.5^x,$$
$$\frac{d}{dx}\left(3^x\right) \approx (1.10)\,3^x.$$

1. Hopefully yesterday, you found that 2.7 got closer to L approaching 1. Try to find one more digit, the digit in the hundredth's place.

It turns out that you can keep testing L values that get closer to 1 and get more and more digits of a very special number in mathematics that we call Euler's number, or e.

The number ***e*** is the value of a that satisfies the equation

$$\lim_{h \to 0} \frac{a^h - 1}{h} = 1, \tag{5}$$

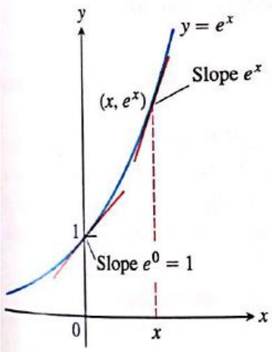
so that

$$\frac{d}{dx}(e^x) = 1 \cdot e^x = e^x. \tag{6}$$

2. This is a big deal! There is a function whose slope at every point is equal to the function evaluated at that same point. Try it out. Graph $y = e^x$ on a graphing calculator or Desmos and find the slope at any point.

To 15 decimal places,

$$e = 2.7\ 1828\ 1828\ 45\ 90\ 45$$



6.4 At each x, the slope of the curve $y = e^x$ is e^x.

The Geometric Significance of $\lim_{h\to 0} \frac{a^h - 1}{h}$

You may have noticed that the graph of each of the functions $f(x) = a^x$ passes through the point (0, 1) with a different slope. And what is this slope? It is none other than

$$f'(0) = \lim_{h\to 0} \frac{a^{0+h} - a^0}{h} = \lim_{h\to 0} \frac{a^h - 1}{h}, \tag{8}$$

the limit at the end of Eq. (1). Thus, Eq. (1) can be reinterpreted to say that if $f(x) = a^x$, then

$$f'(x) = f'(0) \cdot f(x). \tag{9}$$

The derivative of $y = a^x$ is a^x multiplied by the slope at the point where the graph crosses the y-axis. The graph of e^x crosses the y-axis with slope 1, so e^x is its own derivative (Fig. 6.4).

3. Use the guide above to find the slope at the following points.

x = 0:

x = 1:

x = 2:

Now what if we wanted to take the derivative of e^{5x}? We can, and we'd have to use the chain rule as follows:

If u is a differentiable function of x, then

$$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}.$$

So d/dx (e^{5x}) = 5 e^{5x}.

4. Now try these examples and check your answers on p. 351.

$$\frac{d}{dx}(e^{kx})$$

$$\frac{d}{dx}(e^{-x})$$

$$\frac{d}{dx}(e^{x^2})$$

$$\frac{d}{dx}e^{\sin x}$$

On a separate sheet of paper, do the following Exercises on p. 353:
7, 9, 27, 29, 31, and 33.

Friday, May 22

Calculus Unit: Quarter Review
Lesson 5: Minor Assessment

Take a few minutes to look over your review work, then flip over and take your last minor assessment of the year in calculus!

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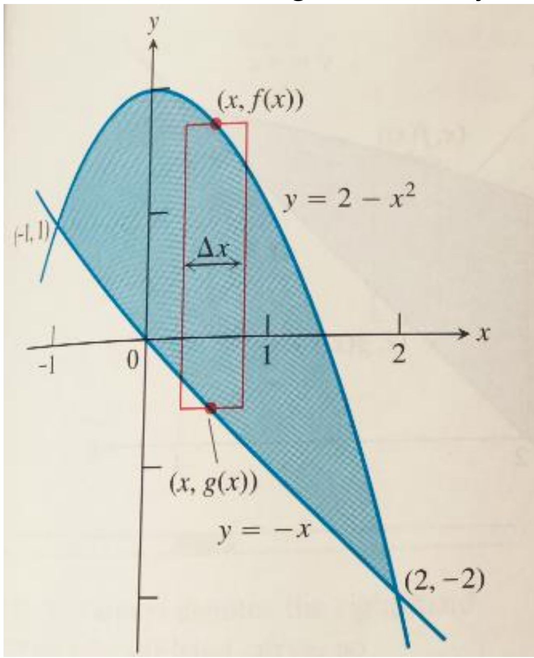
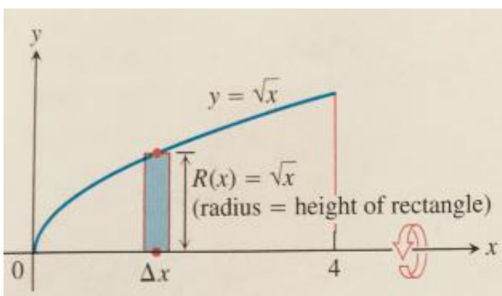
Calculus – Minor Assessment

Name: ___________________________

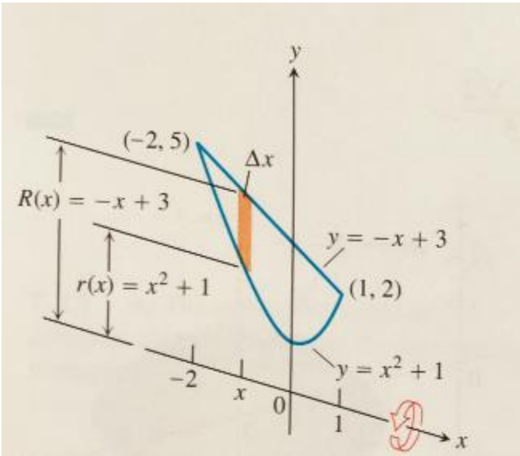
1. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.



2. The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$, and the x-axis is revolved about the x-axis to generate the solid as seen below. Find its volume.



3. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.



4. Write 2-3 complete sentences explaining what Euler's number is, how to find it, and why it's important.

Take the derivative of the following functions.

5. y = e-5x

6. $y = e^{3x^2}$

7. $y = xe^x$