Name: ____________________

Algebra 1

May 11 – 15

Time Allotment: 40 minutes per day

Zoom Guided Instruction	Day/Time this Week
1st Period Ms. Steger	Monday & Wednesday, 10:00 – 10:50am
2nd Period Ms. Steger	Monday & Wednesday, 11:00 – 11:50am
3rd Period Ms. Brintnall	Monday & Wednesday, 1:00 – 1:50pm
4th Period Ms. Brintnall	Tuesday & Thursday, 10:00 – 10:50am

Student Name: ________________________________

Teacher Name (**circle one**): Steger Brintnall

Teacher emails: Vanessa.steger@greatheartsnorthernoaks.org and melanie.brintnall@greatheartsnorthernoaks.org. Ms. Brintnall will be teaching Mrs. Chubb's Algebra 1 class for the remainder of the school. If you were in Mrs. Chubb's class, you should email Ms. Brintnall for help if needed!

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.	I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.
Student signature:	*Parent signature:*
________________________	________________________

GreatHearts®

Algebra 1
May 11 – May 15

Name: ____________________

Packet Overview

Date	Objective(s)	Page Number
Monday, May 11	Differentiate between linear, quadratic, and exponential functions based on their graphs.	3 – 5
Tuesday, May 12	Identify the equation for an exponential function.	6 – 7
Wednesday, May 13	Write a function that models exponential growth.	8 – 10
Thursday, May 14	Write a function that models exponential decay.	11 – 12
Friday, May 15	Represent an exponential problem with an equation and graph.	13 – 15
	Answer key for daily work	

Dear Algebra 1 students,

Last week we finished up quadratic functions. In honor of our last weeks of the 2019-2020 school year, we have a final type of function to talk about in Algebra 1. We won't even tell you the name quite yet… though you might be able to guess it from the Objectives table above. 😊

A couple of notes:

- You can use a CALCULATOR on this packet!!
- AFTER you write an answer for a problem, check the answer key right away to make sure that you don't keep moving until you understand that problem.
- Thanks to https://www.engageny.org/, author and illustrator David Barry and Donna Perrone, and website https://www.ck12.org/book/ck-12-algebra-i-second-edition/section/8.7/ for some of the problems in this week's packet.

We would love to keep you seeing you in Zoom Instruction! Have a great week,

Ms. Steger and Ms. Brintnall

Algebra 1
May 11 – May 15

Name: ____________________

Monday, May 4

Lesson 1 objective: Differentiate between linear, quadratic, and exponential functions based on their graphs.

Two equipment rental companies have different penalty policies for returning a piece of equipment late.

Company 1	**Company 2**
On day 1, the penalty is \$5. On day 2, the penalty is \$10. On day 3, the penalty is \$15. On day 4, the penalty is \$20, and so on, increasing by \$5 each day the equipment is late.	On day 1, the penalty is \$0.01. On day 2, the penalty is \$0.02. On day 3, the penalty is \$0.04. On day 4, the penalty is \$0.08, and so on, doubling in amount each additional day late.

Jim rented a digger from Company 2 because he thought it had the better late return policy. The job he was doing with the digger took longer than he expected, but it did not concern him because the late penalty seemed so reasonable. When he returned the digger 15 days late, he was shocked by the penalty fee. **Fill out the tables below to find out what did he paid, and what would he have paid if he had used Company 1 instead.**

Company 1	
Day	**Penalty**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

Company 2	
Day	**Penalty**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

1. How did the amount of the late charge change from day to day in Company 1?

____

____

2. How did the amount of the late charge change from day to day in Company 2?

____

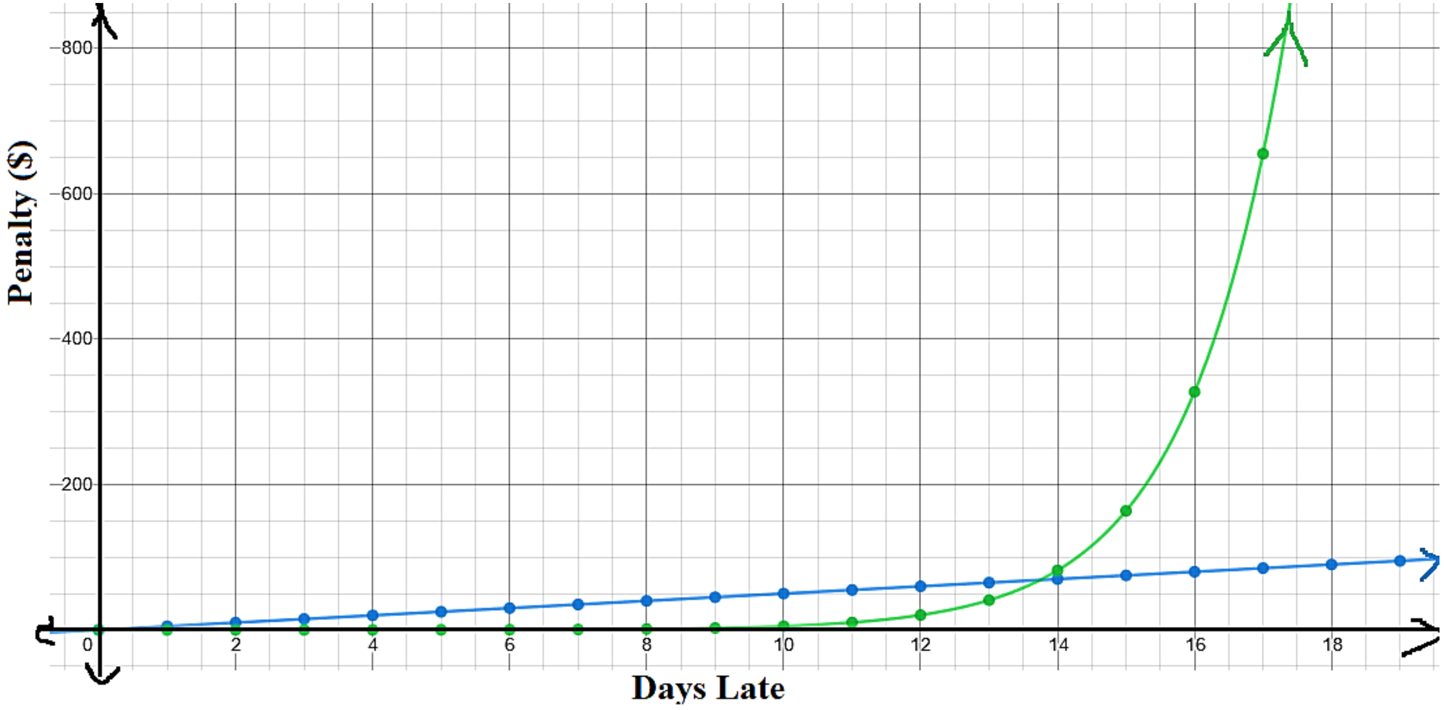
____

3. How much would the late charge have been after **20** days under Company 2? Show your work in the space on the right of the tables above and **write your answer on this blank:** ________________

Algebra 1

May 11 – May 15 **Name:** ______________________

Now isn't that just crazy?? Thousands of dollars for 20 days late when you started with a 1¢ penalty! Let's see this same information represented on a graph:



4. Does the straight line represent Company 1 or Company 2? ____________________
5. What is the *slope* (also called *rate of change*) of the straight line? _______________
6. What do you notice about the *rate of change* for the Company 2 graph?

____

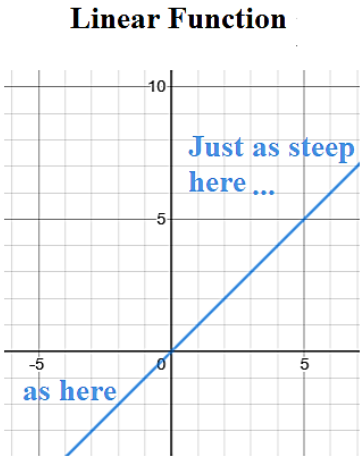
____

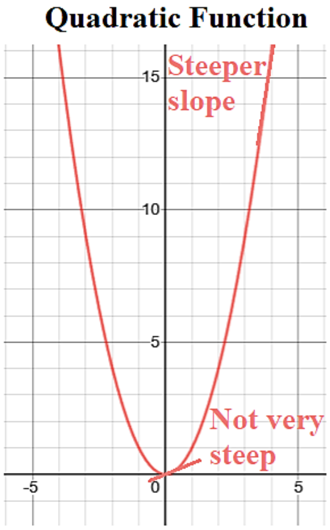
7. **MARK** the place where to two curves intersect. This is the place where the penalty is the same for both Company 1 and Company 2. If Jim was 13 days late, would he have been better off with Company 1 or Company 2? How does the graph tell you this?

____

____

Linear functions have *constant rates of change* (their slopes don't change no matter where you are on the graph). In our last unit, we also talked about *quadratic functions.* Quadratic functions curve (they are not straight) so the slope DOES change depending on where you are on the graph:





Algebra 1
May 11 – May 15 **Name: ______________________**

In our functions of the week, we often represent real-life situations with equations and graphs. For example, last week we looked at a toy rocket launching into the air and realized that its path could be modeled with a quadratic (2nd degree) equation $h(t) = -16t^2 + 128t$. This function is a transformed version of the parent quadratic function $y = x^2$ and $h(t)$ shares a lot of the same properties as the parent quadratic function (it has a vertex, it is symmetrical, the axis of symmetry goes right through the vertex).

We have talked extensively this year about two types of functions:

- 1st degree **LINEAR functions** like $f(x) = x$, $y = -2x + 1$, and $3x - 7y = 2$.
- 2nd degree **QUADRATIC functions** like $f(x) = x^2$, $y = 2x^2 - 3x + 5$, and $k(x) = -(x+2)^2 - 1$

We are now ending our year of Algebra 1 with a NEW type of function: the **exponential function.** Company 2's late fee policy can be modeled with this type of function, and tomorrow we will figure out what algebraic equation matches with an exponential curve.

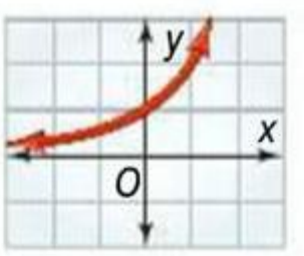
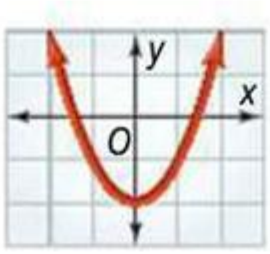
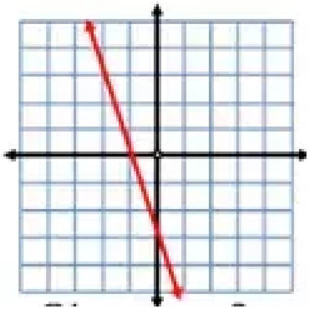
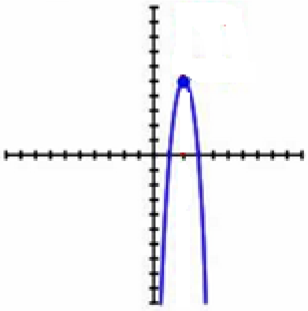
8. Write your own example (different than the ones we gave above) of a linear function: __________________

9. How do we know that $y = -2x + 1$ is a 1st degree function? ____________________________________

10. Write your own example (different than the ones we gave above) of a quadratic function: ________________

11. How do we know that $f(x) = x^2$ is a 2nd degree function? ____________________________________

Label each graph or equation with **linear, quadratic, or exponential:**

12. ______________________ $x + 1 = 2y - 9$	13. ______________________ 	14. ______________________ $y - x^2 = 2x + 3$
15. ______________________ 	16. ______________________ 	17. ______________________ 

Tuesday, May 5

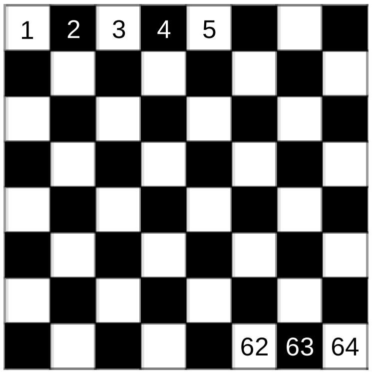
Lesson 2 objective: Identify the equation for an exponential function.

Before jumping in with today's classwork, take a leisurely moment and hop onto Google classroom to watch our video of a book we would have read aloud together in class (perhaps on the steps outside 😊). If you can't watch the video, no worries! This packet gives you all you need.

The Rajah's Rice: A Mathematical Folktale

Imagine the Rajah placing the rice on each of the **64** squares as Chandra requested. We know that the first square is assigned a two grains grain of rice, and each successive square is double the number of grains of rice of the previous square. Fill out the following table for the first 5 squares of rice:

Square #	Grains of Rice
1	**2**
2	
3	
4	
5	



1. As you look from square #1 to 2 to 3 to 4, what operation is happening each time?

____

2. Remember that multiplication is repeated addition. What is repeated multiplication? ______________

3. **We are multiplying by 2 REPEATEDLY.** To think about the equation that will fit this pattern, we are going to use *prime factorization*. Use your factor trees to break up the numbers until you have its prime factorization:

x	y	
Square #	Grains of Rice	Prime factorization:
3	**8**	$\mathbf{2^3}$
4	**16**	
5	**32**	
6	**64**	
7	**128**	
8	**256**	

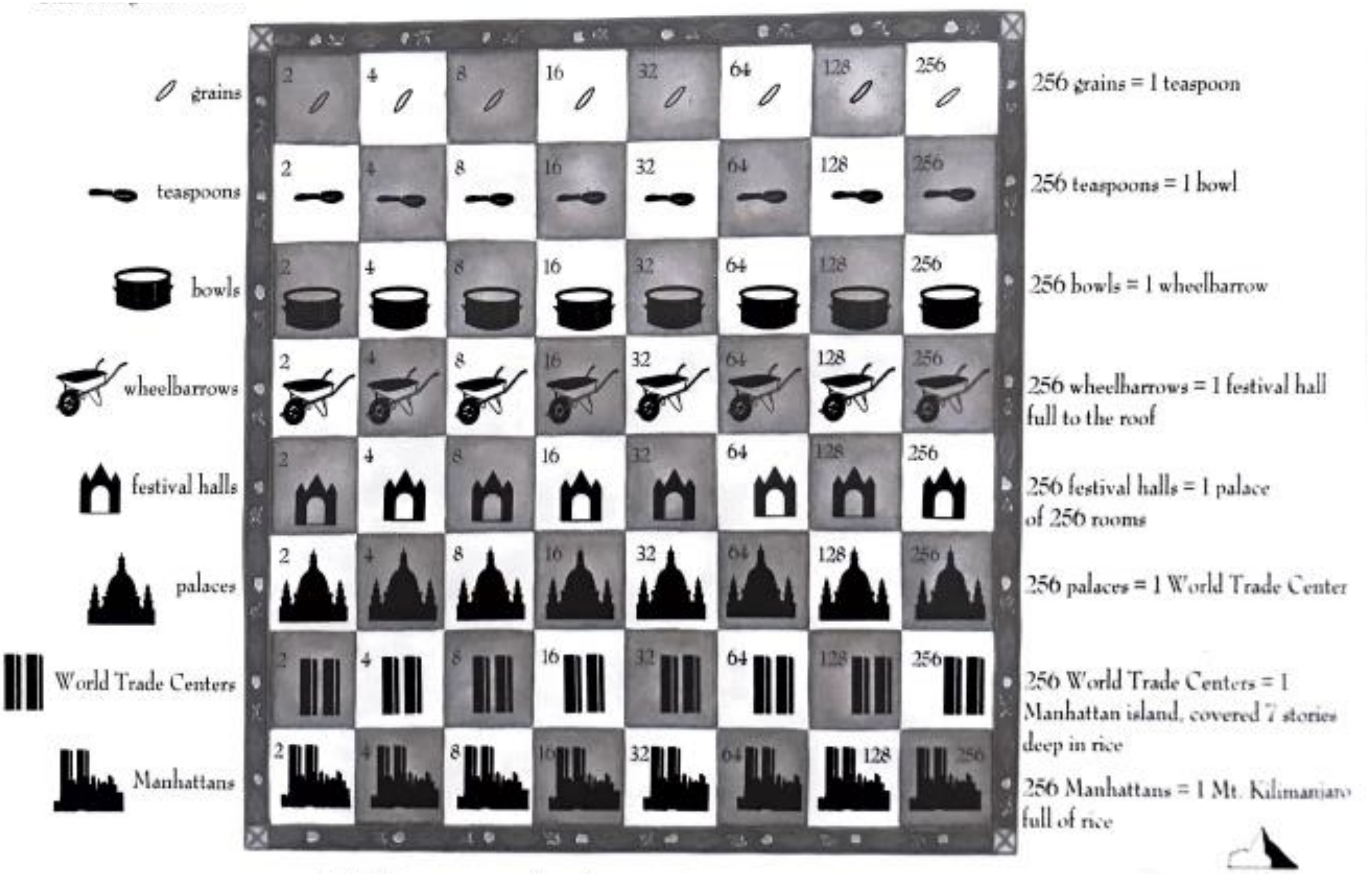
All right, how does the number in the x column seem to be involved in the output (y column)? Try writing an equation that represents this table of values using y and x:

____

4. The prime factorization of our outputs showed us that we have 2 being taken to a certain power. Read the sentences and fill out the blanks: When we were on square #1, 2 was being taken to the first power to get 2. When we were on square #2, 2 was being taken to the second power to get 4. When we were on square #3, 2 was being taken to the third power to get 8. When we were on square #4, 2 was being taken to the ________________ power to get _____. When we were on square ______, 2 was being taken to the fifth power to get _______. When we were on ___________, ___ was being taken to the _______________ power to get ________. Once we are on square #64, what power would 2 be taken to? _________________

Want to know what number that is? It's 18,446,744,073,709,551,616!!! That's
eighteen quintillion,
four hundred and forty-six quadrillion,
seven hundred and forty four trillion,
seventy three billion,
seven hundred and nine million,
five hundred and fifty one thousand,
six hundred and sixteen!!!!!!!!!!!

And THAT's the power of exponential growth.



Add all 64 squares together and you get India, covered knee deep in rice.

Imagine how many grains Chandra would have if there were just *one more square* on the chess board. Pretty crazy right???? We hope you enjoyed today's story and the math along with it. Go ask someone what they think 2 times itself 64 times might be – see if they guess that it's in the quintillions! Have a great day everyone!

Algebra 1
May 11 – May 15

Name: ____________________

Wednesday, May 6

Lesson 3 Objective: Write a function that models exponential growth.

A typical thickness of a newspaper is 0.001 cm. This seems pretty thin, right? Let's see what happens when we start folding the newspaper. Optional: With your parents' permission, you can watch a visualization of this with the **"Exponential Growth: How Folding Paper Can Get You to the Moon"** video that we posted on Google classroom or find it on YouTube.

1. How thick is the stack of paper after 1 fold? After 2 folds? After 5 folds?

2. Pause for a moment and think about how you were calculating your answers for #1. Describe what you would have to do to figure out how thick the stack would be after 45 folds:

 ____

 ____

If we had a FORMULA for this problem, maybe we wouldn't have to do all of that repetitive, tedious multiplication. We could just plug 45 in for x and get our answer. So let's find a formula….

3. Would $f(x) = 2^x$ work for this problem like it did for Chandra's rice problem yesterday? Try plugging in 5 for x into that function and see if your answer matches the one you wrote for #1.

So would $f(x) = 2^x$ be a good model for this problem? (circle one) YES NO

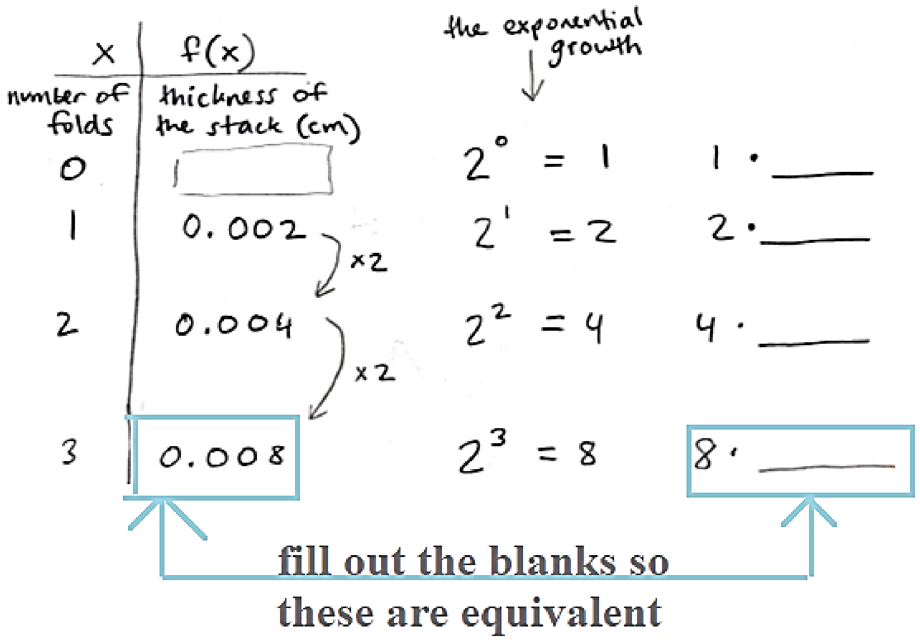
4. Okay, so what makes this problem DIFFERENT from Chandra's rice problem yesterday? Both are doubling with each iteration, so multiplying by 2 is involved somehow… but what makes these problems different?

 ____

 ____

 ____

5. Looking at the pattern in this table of values or at the original problem, how thick is the stack when there are *zero* folds? Fill that in the empty box and then fill out the blanks shown.



x	f(x)		
number of folds	thickness of the stack (cm)	the exponential growth ↓	
0	☐	$2^0 = 1$	1 · ____
1	0.002	$2^1 = 2$	2 · ____
2	0.004	$2^2 = 4$	4 · ____
3	0.008	$2^3 = 8$	8 · ____

fill out the blanks so these are equivalent

So, no matter what row of the table of values we are on, 2^x *is* involved, but it is multiplied by the ORIGINAL THICKNESS each time. Other ways of saying the ORIGINAL THICKNESS are:

- **"the thickness with zero folds" (specific to this problem)**
- **"the output when the input is zero" (in general to all exponentials)**
- **"the y when x is 0" (in general to all exponentials)**
- **"the value of the dependent variable when the independent variable is 0" (in general to all exponentials)**

the number of folds is the exponent

$$\boldsymbol{f(x) = 0.001 \cdot 2^x}$$

Thickness (cm)

Original thickness (cm)

the multiplier that happens with each iteration (in this case doubling)

6. **Let's make sure this equation really models what is happening in this problem. Fill out the empty rows for folds 3 and 4 and say whether they match:**

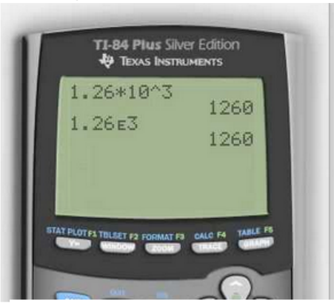
x	f(x) with logic	With formula	Does it match? yes or no
number of folds	thickness of the stack in cm	$f(x)=0.001\cdot2^x$	
0	0.001	$0.001\cdot2^0$ $0.001\cdot1$ 0.001	yes!
1	0.002	$0.001\cdot2^1$ $0.001\cdot2$ 0.002	yes!
2	0.004	$0.001\cdot2^2$ $0.001\cdot4$ 0.004	yes!
3	0.008		
4	0.016		

7. **Okay, since this is a good formula to model this problem, let's calculate how thick the paper stack is after 45 folds.**

This is the function:	We're going to find $f(x)$ when $x = 45$:	So plug this into a calculator:
$\boldsymbol{f(x) = 0.001 \cdot 2^x}$	$\boldsymbol{f(45) = 0.001 \cdot 2^{45}}$	`0.001*2^45`

You may get a crazy number! Write ***exactly*** what you got here: ________________________________

Does your answer have an “E” toward the end? That is the calculator’s way of writing scientific notation:



```
1.26*10^3
                1260
1.26E3
                1260
```

$$\boxed{1.26\text{E}3}$$

really means this:

$$1.26 \times 10^3 = 1.26 \times 1,000 = \boxed{1,260}$$

8. So, according to the Google calculator, $f(45) = 0.001 \cdot 2^{45} =$ 35,184,372,088.8 cm. That’s equal to 351,843,720.888 meters which is equal to 351,843.720888 kilometers. The moon is on average 384,400 kilometers away from the earth. So, how many folds of a newspaper would it take to REACH THE MOON???? *Be specific and explain your answer – don’t just answer with a single number!*

____

____

____

9. According to NASA, the sun is about 149,600,000 kilometers from Earth, which is 14,960,000,000,000 centimeters (my calculator shows this number as `1.496E13`). Use a calculator to fill out the table of values for these REALLY BIG numbers. I showed for the first few what I typed into my TI-83 Plus calculator and what I got out. You should fill out the rest of the chart in a similar way. **With these huge numbers, showing your commas will help you see what number it actually is.**

x	$f(x) = 0.001 \cdot 2^x$ **(thickness in CENTIMETERS)**
46	`0.001*2^46 = 7.036874418E10` **which really means:** $7.036874418 \times 10^{10} = 70,368,744,180$
50	`0.001*2^50 = 1.125899907E12` **which really means:** $1.125899907 \times 10^{12} = 1,125,899,907,000$
51	
52	
53	
54	
55	

I want a thickness that is ___ $\times 10^{13}$, so this is too small

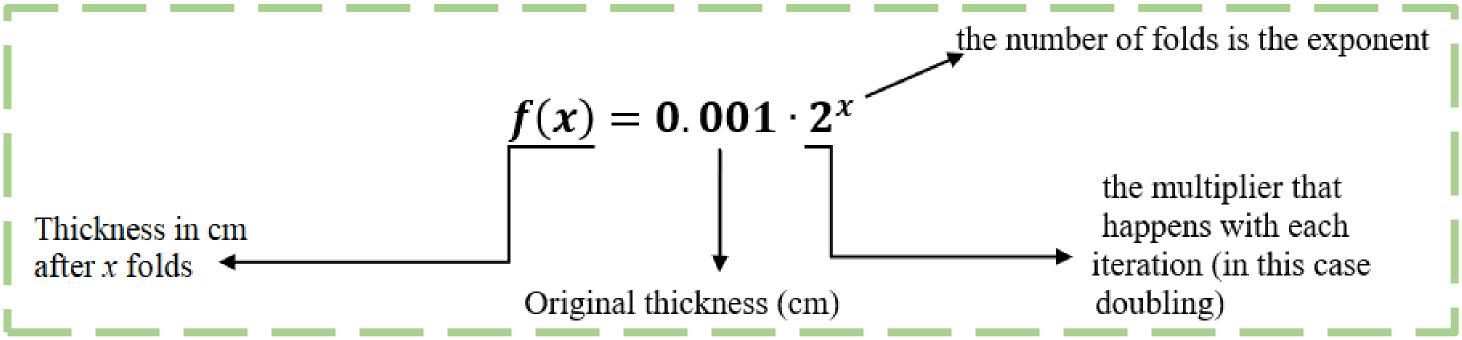
getting closer

10. So how many times would you need to fold a newspaper to reach THE SUN? ___________

Thursday, May 7

Lesson 4 Objective: Write a function that models exponential decay.

Let’s recall the equation we figured out for yesterday’s problem of modeling the thickness of a newspaper stack after a certain number of folds:



With this type of formula in mind, let’s think about a new situation:

For her eighth birthday, Shelley’s grandmother gave her a full bag of candy. Shelley counted her candy and found out that there were 160 pieces in the bag. As you might suspect, Shelley loves candy, so she ate half the candy on the first day. Then her mother told her that if she eats it at that rate, the candy will only last one more day—so Shelley devised a clever plan. She will always eat half of the candy that is left in the bag each day. She thinks that this way she can eat candy every day and never run out.

How much candy does Shelley have at the end of the week? Will the candy really last forever?

1. Let’s make a table of values for this problem. ***Use a calculator as needed.***

x (number of days)	y (number of candy pieces left)
1	How many pieces does Shelley have left after the 1st day?
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	

2. How many candy pieces does Shelley have at zero days? Or, what were Shelley’s ORIGINAL number of candy pieces?

____

3. What number are you repeatedly MULTIPLYING by as you move from Day 1 to Day 2 to Day 3? ________

4. Use your answers from #2 and #3 and the model at the top of the page, what equation could model this situation? Use y as the number of candy pieces left and x as the number of days.

____

5. Do you think Shelley will ever RUN OUT of candy? Why or why not?

____

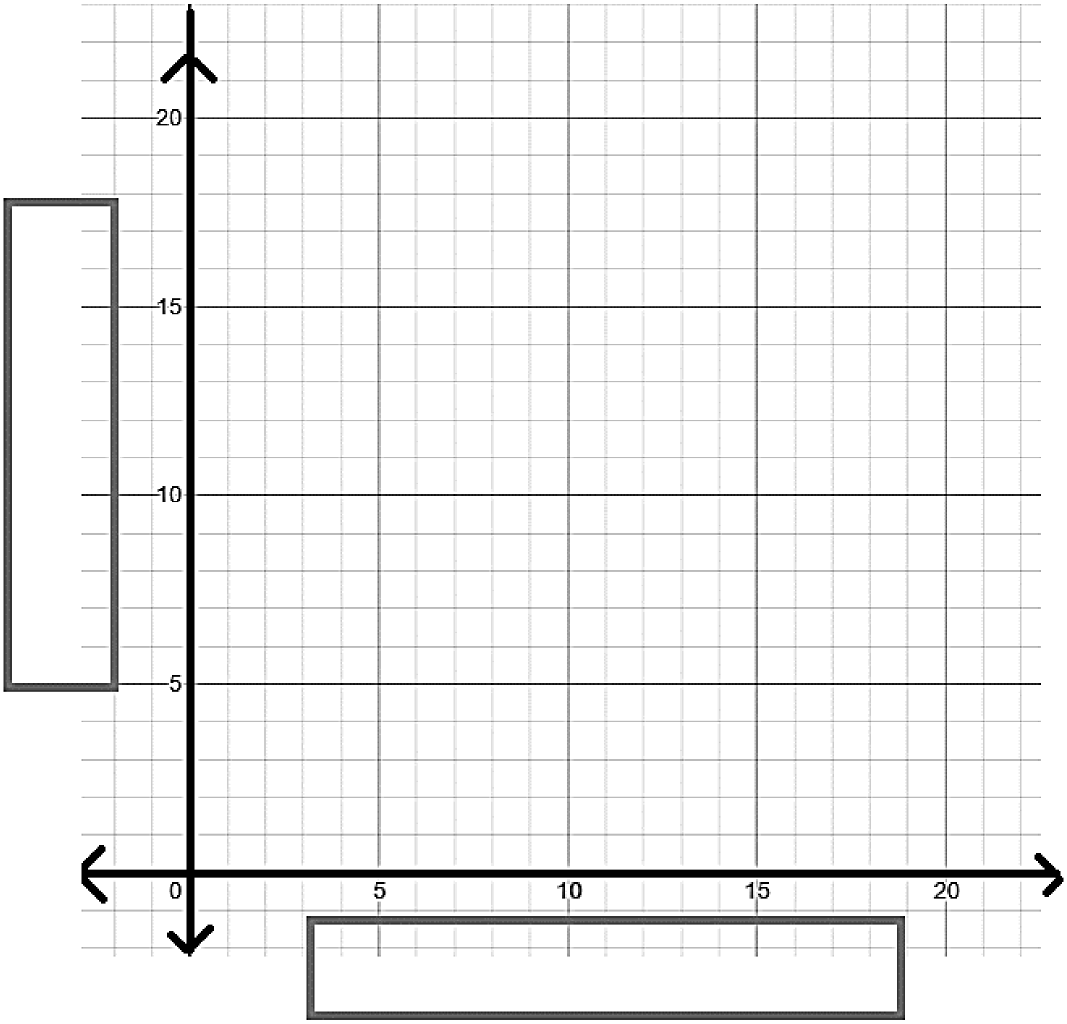
____

____

____

____

6. Now let’s graph this function. Put **LABELS in the boxes by each of the axes, then plot the points you have in your table of values (whichever ones fit onto the graph). Then connect the dots in a nice, smooth curve.**



7. What coordinate is the y-intercept of this function $y = 160 \cdot \left(\frac{1}{2}\right)^x$

(____ , ____)

8. Do you think there is an x-intercept of this graph? Why or why not?

____

____

____

____

____

____

9. Which type of function is this curve? CIRCLE ONE: Linear Quadratic Exponential

10. This function shows EXPONENTIAL DECAY, rather than EXPONENTIAL GROWTH. By decay, we mean that the output (or y-value) is becoming less and less as the input (or x-value) gets bigger and bigger. What part of the function $y = 160 \cdot \left(\frac{1}{2}\right)^x$ makes it a decay function do you think? Why? Remember that the function $f(x) = 0.001 \cdot 2^x$ modeling the paper folding showed GROWTH.

____

____

____

11. Okay, pause for a moment and take a breath. We have talked about an *entirely new* parent function in the last four days. Take a few minutes to look through your packet and respond thoughtfully to the following question. In this question, we are not looking for one exact answer, but rather your thoughts as you reflect on exponential functions: What makes exponential functions so *powerful?*

____

____

Friday, May 8

Lesson 5 Objective: Represent an exponential problem with an equation and graph.

Before we jump in with today's minor assessment, please review on pages 8 and 9 how we came up with equation $\boldsymbol{f(x) = 0.001 \cdot 2^x}$ for the paper folding problem.

1. What was the significance of the 0.001? ____

2. Why is the 2 the only number being taken to the power of x? USE PAGE 8 TO HELP YOU ANSWER!

____

____

Now review how you came up with the equation $\boldsymbol{y = 160 \cdot \left(\frac{1}{2}\right)^x}$ on page 11.

3. What was the significance of the 160? ____

4. Why is the $\frac{1}{2}$ the only number being taken to the power of x?

____

____

Okay, now that you have reminded yourself of these key ideas about exponential functions, let's go to the minor assessment for this week.

<u>Minor Assessment (Quiz)</u>

Please read these boxes carefully before starting on the minor assessment.

- ☐ I understand that I am NOT allowed to use this packet during my quiz.
- ☐ I understand that I am NOT allowed to use my own loose-leaf packet during my quiz.
- ☐ I understand that while Ms. Steger and Ms. Brintnall estimate that the quiz will take 15 minutes, it is okay to spend the time I need.
- ☐ I understand that I am NOT allowed to ask a parent, family member, or friend for help during my quiz.
- ☐ I understand that I am NOT allowed to use the internet or any other resource to help with my quiz.
- ☐ I understand that I **AM ABLE TO USE A CALCULATOR on this quiz.**

Algebra 1
May 11 – May 15

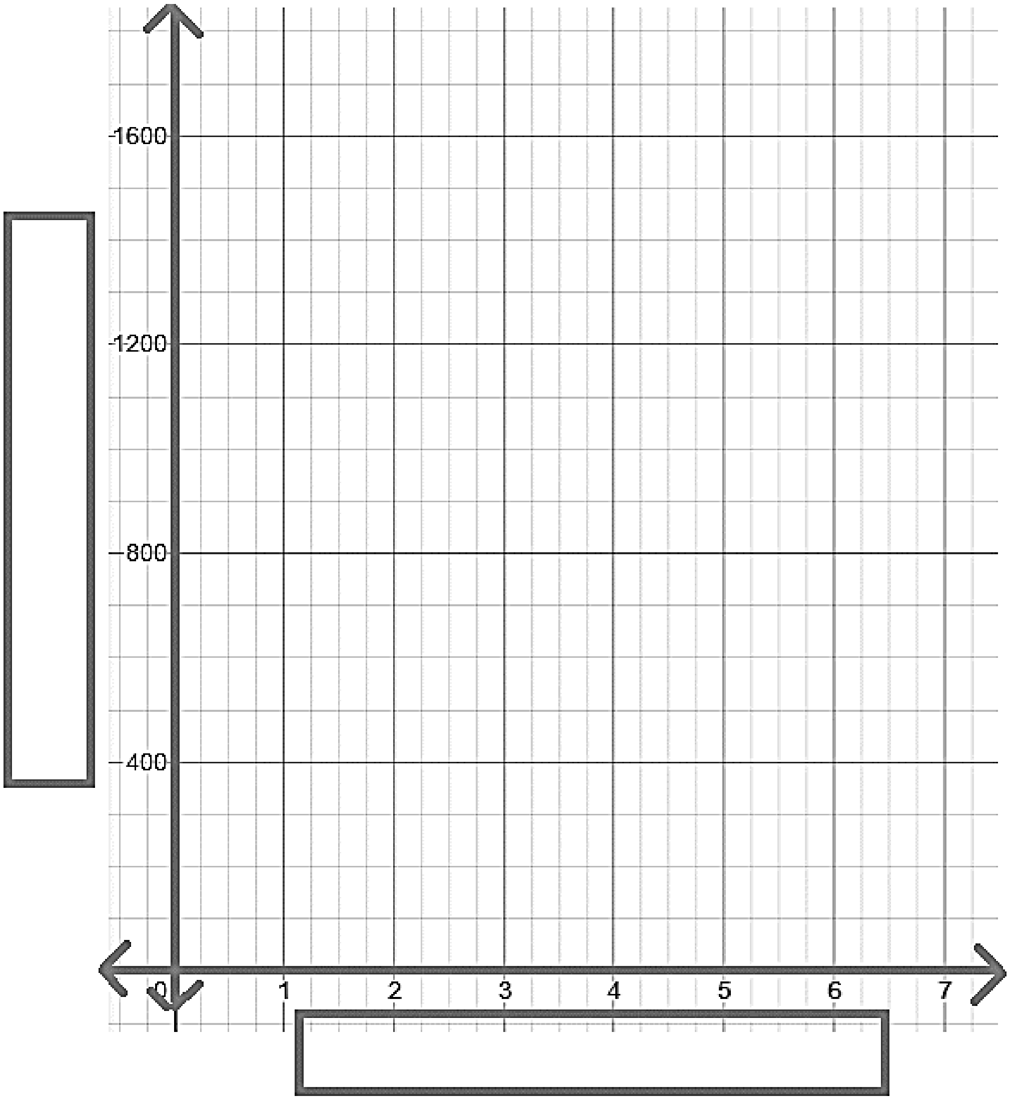
Name: ____________________

Minor Assessment for Week 8

Imagine that you received 2 M&M's on May 1st, but you **tripled** your amount of M&M's every day since then.

1. Complete the following table of values.

t Time in days after May 1st	$f(t)$ Number of M&M's
0	
1	**6**
2	
3	
4	
5	



2. Label the axes and graph the curve using the points in your table of values (or whatever ones you can fit).

3. How many M&M's did you have at $t = 0$?

   ____

4. What number are you multiplying by repeatedly from day 1, to day 2, to day 3? ____________________

5. **Write a function for this table of values using t as the time in days and $f(t)$ as the number of M&M's.** (notice how we're not using x and y in this problem, but the ideas are all the same!)

   ____

6. Is this exponential GROWTH or DECAY? (circle one).

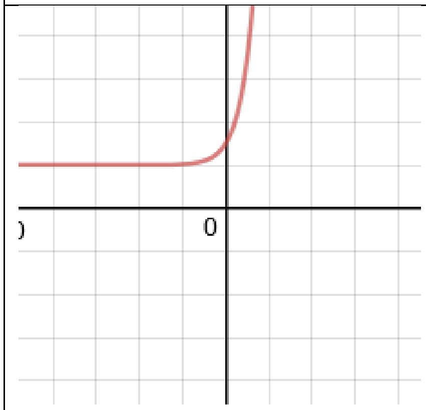
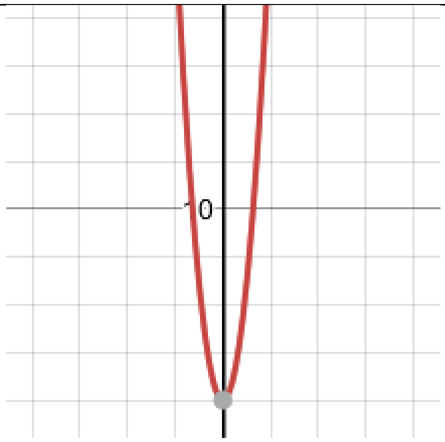
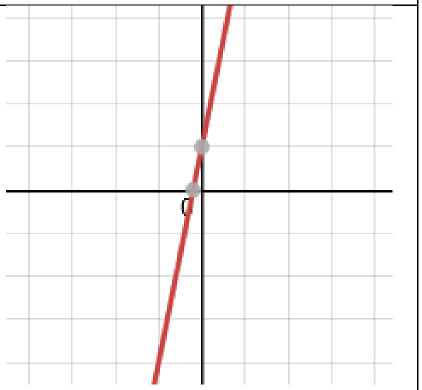
…. Continued on the next page!

CIRCLE which graph matches the equation and then CIRCLE whether the function is linear, quadratic, or exponential.

7. $f(x) = 5x + 2$ **A** **B** **C** linear quadratic exponential

8. $f(x) = 5^x + 2$ **A** **B** **C** linear quadratic exponential

9. $f(x) = 5x^2 + 2$ **A** **B** **C** linear quadratic exponential

A	B	C
		

<u>You are now finished for the week! Remember to upload your pictures of the daily work and minor assessment to the Google Classroom, OR prepare one packet of papers with your name, Algebra 1, and your teacher's name on the very front stapled together to drop off at school. Again, we are checking for following directions when grading! 😊</u>

Algebra 1
May 11 – May 15

Name: ____________________

Answer Key for the Daily Work

Lesson 1
(Monday)

Company 1	
Day	Penalty
1	$5
2	$10
3	$15
4	$20
5	$25
6	$30
7	$35
8	$40
9	$45
10	$50
11	$55
12	$60
13	$65
14	$70
15	$75

s

Company 2	
Day	Penalty
1	$0.01
2	$0.02
3	$0.04
4	$0.08
5	$0.16
6	$0.32
7	$0.64
8	$1.28
9	$2.56
10	$5.12
11	$10.24
12	$20.48
13	$40.96
14	$81.92
15	$163.84

1. Free response
2. Free response
3. $5,242.88
4. Company 1
5. 5 dollars per day
6. Free response
7. Company 2 – at 13 days its curve is below the Company 1 curve, showing that its penalty is less.
8. Free response
9. The highest exponent on any of the variables is 1.
10. Free response
11. The highest exponent on any of the variables is 2.
12. Linear
13. Exponential
14. Quadratic
15. Quadratic
16. Linear
17. Quadratic

Lesson 2
Tuesday

1. Multiplying by 2
2. Exponents
3. $2^4, 2^5, 2^6, 2^7, 2^8 \quad y = 2^x$
4. Fourth, 16, 5, 32, square #6, 2, sixth, 64, 64th power

Lesson 3
Wednesday

1. 0.002 cm, 0.004 cm, 0.032 cm
2. Free response
3. $f(5) = 2^5 = 32$ which doesn't match – $f(x) = 2^x$ is not a good model for this problem.
4. Free response

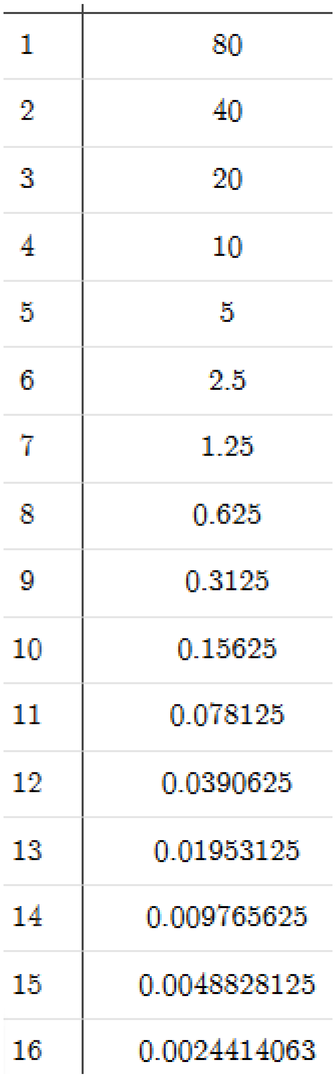
5. 0.001
6. Yes they match!
7. Answer given on page 10
8. Between 45 and 46 folds – your answer should have more detail though!
9. Depending on your calculator, your answers may have the same first several digits but our answers may be different toward the end:

$$\begin{gathered}2{,}251{,}799{,}814{,}000\\4{,}503{,}599{,}627{,}000\\9{,}007{,}199{,}255{,}000\\18{,}014{,}398{,}510{,}000\\36{,}028{,}797{,}020{,}000\end{gathered}$$

10. Between 53 and 54 folds.

Lesson 4
Thursday

1.



1	80
2	40
3	20
4	10
5	5
6	2.5
7	1.25
8	0.625
9	0.3125
10	0.15625
11	0.078125
12	0.0390625
13	0.01953125
14	0.009765625
15	0.0048828125
16	0.0024414063

2. 160
3. $\frac{1}{2}$
4. $y = 160 \cdot \left(\frac{1}{2}\right)^x$
5. Free response
6. The graph:

7. (0,160)
8. No – the line gets closer and closer to $y = 0$ but never actually reaches it.
9. Exponential
10. Multiplying by $\frac{1}{2}$ makes a number smaller (causing exponential decay) while multiplying by 2 makes a number larger (causing exponential growth).
11. Free response

Lesson 5
(Friday)

1. It's the original thickness (the y when $x = 0$)
2. Each time you fold the paper, the previous value is multiplied by 2.
3. Provide this answer yourself.
4. Provide this answer yourself.