Pre-Calculus: Week of May 11 – May 15

Time Allotment: 40 minutes per day

Student Name: ________________________________

Teacher Name: Mrs. Melisa R. Walters

Packet Overview

Date	Objective(s)	Page Number
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Zoom Guided Instruction:

Period 3 Monday and Wednesday from 1:00PM – 1:50 PM
Period 4 Tuesday and Thursday from 10:00AM – 10:50 AM
Period 6 Tuesday and Thursday from 1:00PM – 1:50 PM

Thank you for your hard work students. I appreciate all of you. Have a great day!

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

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I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

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Monday, May 11

Pre-Calculus: Chapter 11
Lesson 1: Geometric Sequences

Objective: Find any term of a Geometric Sequence using Recursive Formulas

Lesson 1
A recursive formula allows us to find any term of a geometric sequence by using the previous term. Each term is the product of the common ratio and the previous term. For example, suppose the common ratio is 9. Then each term is nine times the previous term. As with any recursive formula, the initial term must be given.

recursive formula for a geometric sequence
The recursive formula for a geometric sequence with common ratio r and first term a_1 is

$$a_n = ra_{n-1}, n \geq 2$$

STEPS on how to:
Given the first several terms of a geometric sequence, write its recursive formula.

1. State the initial term.
2. Find the common ratio by dividing any term by the preceding term.
3. Substitute the common ratio into the recursive formula for a geometric sequence.

Example: Using Recursive Formulas for Geometric Sequences
Write a recursive formula for the following geometric sequence. {6, 9, 13.5, 20.25, ... }

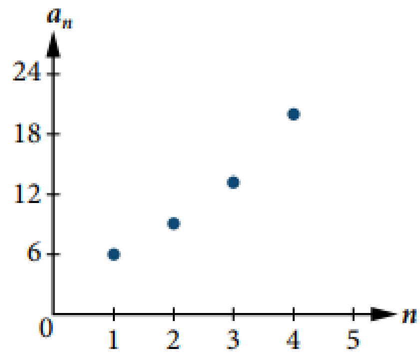
Solution: The first term is given as 6. The common ratio can be found by dividing the second term by the first term.

$$r = \frac{9}{6} = 1.5$$

Substitute the common ratio into the recursive formula for geometric sequences and define a_1.

$$a_n = ra_{n-1}$$
$$a_n = 1.5a_{n-1} \text{ for } n \geq 2$$
$$a_1 = 6$$

Analysis The sequence of data points follows an exponential pattern. The common ratio is also the base of an exponential function as shown below.



Question: Do we have to divide the second term by the first term to find the common ratio?

Answer: No. We can divide any term in the sequence by the previous term. It is, however, most common to divide the second term by the first term because it is often the easiest method of finding the common ratio.

Your turn to try it!
Write a recursive formula for the following geometric sequence. $\left\{2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots\right\}$

Any questions right now? Please email me at Melisa.Walters@greatheartsnorthernoaks.org

Exercises for Monday, May 11, 2020

Section Exercises page 967 #25, 27

Tuesday, May 12

Pre-Calculus: Chapter 11
Lesson 2: Geometric Sequences

Objective: Calculating Geometric Sequences using the Explicit Formulas

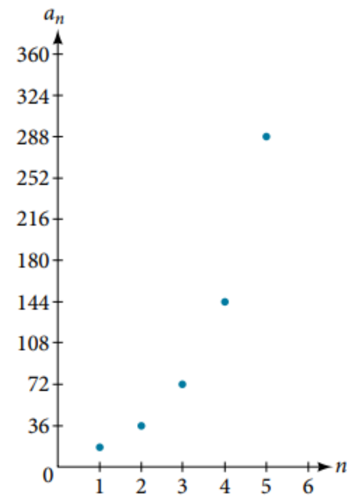
Lesson 2 Using Explicit Formulas for Geometric Sequences
Because a geometric sequence is an exponential function whose domain is the set of positive integers, and the common ratio is the base of the function, we can write explicit formulas that allow us to find particular terms.

$$a_n = a_1 * r^{n-1}$$

Let's take a look at the sequence $\{18, 36, 72, 144, 288, ...\}$. This is a geometric sequence with a common ratio of 2 and an exponential function with a base of 2. An explicit formula for this sequence is

$$a_n = 18 * 2^{n-1}$$

The graph of the sequence:



explicit formula for a geometric sequence
The nth term of a geometric sequence is given by the explicit formula:

$$a_n = a_1 r^{n-1}$$

Example: Writing Terms of Geometric Sequences Using the Explicit Formula
Given a geometric sequence with $a_1 = 3$ and $a_4 = 24$, find a_2 .

Solution: The sequence can be written in terms of the initial term and the common ratio r.

$$3, 3r, 3r^2, 3r^3, ...$$

Find the common ratio using the given fourth term.

$a_n = a_1 r^{n-1}$	
$a_4 = 3r^3$	Write the fourth term of sequence in terms of a_1 and r
$24 = 3r^3$	Substitute 24 for a_4
$8 = r^3$	Divide
$r = 2$	Solve for the common ratio

Find the second term by multiplying the first term by the common ratio.

$$\begin{aligned} \boldsymbol{a_2} &= \boldsymbol{2a_1} \\ &= 2(3) \\ &= 6 \end{aligned}$$

Analysis The common ratio is multiplied by the first term once to find the second term, twice to find the third term, three times to find the fourth term, and so on. The tenth term could be found by multiplying the first term by the common ratio nine times or by multiplying by the common ratio raised to the ninth power.

YOUR turn to try it!

Given a geometric sequence with $a_2 = 4$ and $a_3 = 32$, find a_6.

****Please check answer key before proceeding.*

Example: Writing an Explicit Formula for the nth Term of a Geometric Sequence
Write an explicit formula for the nth term of the following geometric sequence.

$$\{2, 10, 50, 250,...\}$$

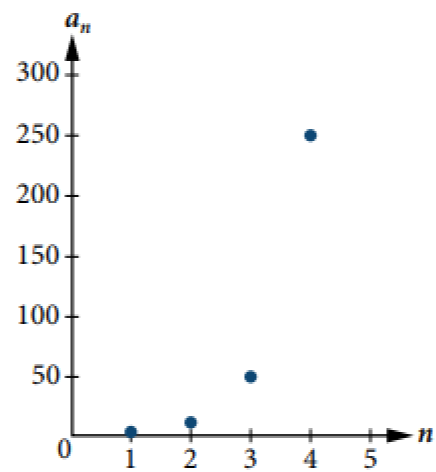
Solution: The first term is 2. The common ratio can be found by dividing the second term by the first term

$$\frac{10}{2} = 5$$

The common ratio is 5. Substitute the common ratio and the first term of the sequence into the formula.

$$a_n = a_1 r^{(n-1)}$$
$$a_n = 2 \cdot 5^{n-1}$$

The graph of this sequence shows an exponential pattern



Try it!

Write an explicit formula for the following geometric sequence. {–1, 3, –9, 27, ...}

****Please check answer key before proceeding*

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Exercises for Tuesday, May 12, 2020

Section Exercises page 968 #35

Wednesday, May 13

Pre-Calculus: Chapter 11
Lesson 3: Geometric Sequences

Objective: Solving Application Problems with Geometric Sequences

Lesson 3

In real-world scenarios involving arithmetic sequences, we may need to use an initial term of a0 instead of a1 . In these problems, we can alter the explicit formula slightly by using the following formula:

$$a_n = a_0 r^n$$

Example:

In 2013, the number of students in a small school is 284. It is estimated that the student population will increase by 4% each year.

a. Write a formula for the student population.
b. Estimate the student population in 2020.

Solution:

a. The situation can be modeled by a geometric sequence with an initial term of 284. The student population will be 104% of the prior year, so the common ratio is 1.04. Let P be the student population and n be the number of years after 2013. Using the explicit formula for a geometric sequence we get
$$P_n = 284 * 1.04^n$$
b. We can find the number of years since 2013 by subtracting. $2020 - 2013 = 7$
We are looking for the population after 7 years. We can substitute 7 for n to estimate the population in 2020.
$$P_7 = 284 * 1.04^7 \approx 374$$

The student population will be about 374 in 2020

YOUR Turn to Try it!
A business starts a new website. Initially the number of hits is 293 due to the curiosity factor. The business estimates the number of hits will increase by 2.6% per week.

a. Write a formula for the number of hits.
b. Estimate the number of hits in 5 weeks.

****Please check the answer key before proceeding.*

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<u>Exercise for Wednesday, May 13, 2020</u>

Section Exercises page 968 #49

<u>Thursday, May 14</u>

Pre-Calculus: Chapter 11
Lesson 4: Series and Their Notation

Objective: Solve annuity problems using summation notation

Lesson 4:
A couple decides to start a college fund for their daughter. They plan to invest $50 in the fund each month. The fund pays 6% annual interest, compounded monthly. How much money will they have saved when their daughter is ready to start college in 6 years? In this section, we will learn how to answer this question. To do so, we need to consider the amount of money invested and the amount of interest earned.

<u>Using Summation Notation</u>
To find the total amount of money in the college fund and the sum of the amounts deposited, we need to add the amounts deposited each month and the amounts earned monthly. The sum of the terms of a sequence is called a series.
Consider, for example, the following series.

$$3 + 7 + 11 + 15 + 19 + \ldots$$

The **nth partial sum** of a series is the sum of a finite number of consecutive terms beginning with the first term. The notation Sn represents the partial sum.

$$S_1 = 3$$
$$S_2 = 3 + 7 = 10$$
$$S_3 = 3 + 7 + 11 = 21$$
$$S_4 = 3 + 7 + 11 + 15 = 36$$

Summation notation is used to represent series. Summation notation is often known as sigma notation because it uses the Greek capital letter sigma, Σ, to represent the sum. Summation notation includes an explicit formula and specifies the first and last terms in the series. An explicit formula for each term of the series is given to the right of the sigma. A variable called **the index of summation** is written below the sigma. The index of summation is set equal to the **lower limit of summation**, which is the number used to generate the first term in the series. The number above the sigma, called **the upper limit of summation**, is the number used to generate the last term in a series.

Upper limit of summation ⟶ 5

$$\sum_{k=1}^{5} 2k \longleftarrow \text{Explicit formula for } k\text{th term of series}$$

Index of summation ⟶ $k = 1$ ⟵ Lower limit of summation

If we interpret the given notation, we see that it asks us to find the sum of the terms in the series $a_k = 2k$ for k = 1 through k = 5. We can begin by substituting the terms for k and listing out the terms of this series.

$$a_1 = 2(1) = 2$$
$$a_2 = 2(2) = 4$$
$$a_3 = 2(3) = 6$$
$$a_4 = 2(4) = 8$$
$$a_5 = 2(5) = 10$$

We can find the sum of the series by adding the terms:

$$\sum_{k=1}^{5} 2k = 2 + 4 + 6 + 8 + 10 = 30$$

summation notation

The sum of the first n terms of a **series** can be expressed in **summation notation** as follows:

$$\sum_{k=1}^{n} a_k$$

This notation tells us to find the sum of a_k from $k = 1$ to $k = n$.

k is called the **index of summation**, 1 is the **lower limit of summation**, and n is the **upper limit of summation**.

Question: Does the lower limit of summation have to be 1?

Answer: No. The lower limit of summation can be any number, but 1 is frequently used. We will look at examples with lower limits of summation other than 1.

<u>STEPS on how to</u>

Given summation notation for a series, evaluate the value.

1. Identify the lower limit of summation.
2. Identify the upper limit of summation.
3. Substitute each value of k from the lower limit to the upper limit into the formula.
4. Add to find the sum.

Example: Using Summation Notation
Evaluate

$$\sum_{k=1}^{7} k^2$$

Solution: According to the notation, the lower limit of summation is 3 and the upper limit is 7. So we need to find the sum of k^2 from k = 3 to k = 7. We find the terms of the series by substituting k = 3, 4, 5, 6, and 7 into the function k^2.

We add the terms to find the sum.

$$\begin{aligned}\sum_{k=3}^{7} k^2 &= 3^2 + 4^2 + 5^2 + 6^2 + 7^2 \\ &= 9 + 16 + 25 + 36 + 49 \\ &= 135\end{aligned}$$

Your Turn to try it!

Evaluate $\sum_{k=2}^{5} (3k - 1)$.

**Please check answer key*

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

<u>Exercises for Thursday May 14, 2020</u>

Section Exercises page 979 #7, 9

Friday, May 15

Pre-Calculus: Chapter 11
Lesson 5: Series

Objective: Calculating Arithmetic Series

****NOTE: First, please complete the lesson then you may take the minor assessment located at the end of the packet. The minor assessment will need to be scanned in to the minor assessment submission on google classroom.*

Lesson 5: Using the Formula for Arithmetic Series

Just as we studied special types of sequences, we will look at special types of series. Recall that an arithmetic sequence is a sequence in which the difference between any two consecutive terms is the common difference, d. The sum of the terms of an **arithmetic sequence** is called an arithmetic series. We can write the sum of the first n terms of an arithmetic series as:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_n - d) + a_n \,.$$

We can also reverse the order of the terms and write the sum as

$$S_n = a_n + (a_n - d) + (a_n - 2d) + \ldots + (a_1 + d) + a_1 \,.$$

If we add these two expressions for the sum of the first *n* terms of an arithmetic series, we can derive a formula for the sum of the first *n* terms of any arithmetic series.

$$\begin{array}{rl} S_n = & a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_n - d) + a_n \\ + \; S_n = & a_n + (a_n - d) + (a_n - 2d) + \ldots + (a_1 + d) + a_1 \\ \hline 2S_n = & (a_1 + a_n) + (a_1 + a_n) + \ldots + (a_1 + a_n) \end{array}$$

Because there are n terms in the series, we can simplify this sum to

$$2S_n = n(a_1 + a_n).$$

We divide by 2 to find the formula for the sum of the first *n* terms of an arithmetic series

$$S_n = \frac{n(a_1 + a_n)}{2}$$

formula for the sum of the first n terms of an arithmetic series
An **arithmetic series** is the sum of the terms of an arithmetic sequence. The formula for the sum of the first n terms of an arithmetic sequence is

$$S_n = \frac{n(a_1 + a_n)}{2}$$

STEPS on how to:
Given terms of an arithmetic series, find the sum of the first n terms.

1. Identify a_1 and a_n .
2. Determine *n*.
3. Substitute values for a_1, a_n , and *n* into the formula $S_n = \frac{n(a_1+a_n)}{2}$
4. Simplify to find S_n.

Example: Finding the First n Terms of an Arithmetic Series
Find the sum of each arithmetic series.

a. 5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32
b. 20 + 15 + 10 + … + −50
c.

$$\sum_{k=1}^{12} 3k - 8$$

Solution:

a. We are given $a_1 = 5$ and $a_n = 32$.
Count the number of terms in the sequence to find n = 10.
Substitute values for a_1, a_n, and *n* into the formula and simplify.

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_{10} = \frac{10(5 + 32)}{2} = 185$$

b. We are given $a_1 = 20$ and $a_n = -50$.
Use the formula for the general term of an arithmetic sequence to find n.

$$a_n = a_1 + (n-1)d$$
$$-50 = 20 + (n-1)(-5)$$
$$-70 = (n-1)(-5)$$
$$14 = n - 1$$
$$15 = n$$

Substitute values for a_1, a_n, n into the formula and simplify

$$S_n = \frac{n(a_1 + a_n)}{2}$$
$$S_{15} = \frac{15(20 - 50)}{2} = -225$$

c. To find a_1, substitute k = 1 into the given explicit formula.

$$a_k = 3k - 8$$
$$a_1 = 3(1) - 8 = -5$$

We are given that $n = 12$. To find a_{12}, substitute $k = 12$ into the given explicit formula.

$$a_k = 3k - 8$$
$$a_{12} = 3(12) - 8 = 28$$

Substitute values for a_1, a_n, and n into the formula and simplify.

$$S_n = \frac{n(a_1 + a_n)}{2}$$
$$S_{12} = \frac{12(-5 + 28)}{2} = 138$$

Your turn to Try It!

Use the formula to find the sum of the arithmetic series. 13 + 21 + 29 + … + 69

**Please check answer key*

Exercises for Friday, May 15, 2020

Section Exercises page 980 #31, 33, 35

ANSWER KEY

Exercises for Monday, May 11, 2020

Try it! $a_1 = 2;\ a_n = \frac{2}{3}a_{n-1}$ for $n \geq 2$

Answers: **25.** $a = -32,\ a_n = \frac{1}{2}a_{n-1}$ **27.** $a_1 = 10,\ a_n = -0.3\,a_{n-1}$

Exercises for Tuesday May 12, 2020

Try it! $a_6 = 16{,}384$
Try it! $a_n = -(-3)^{n-1}$

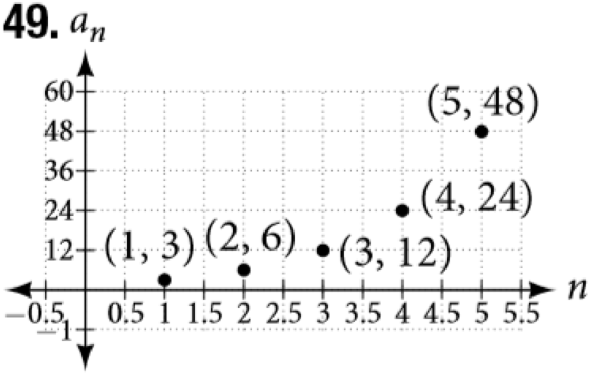
Answers: 35. $a_n = 3^{n-1}$

Exercises for Wednesday May 13, 2020

Try it! a. $P_n = 293 * 1.026a^n$ b. The number of hits will be 333.

Exercises:

49.



Exercises for Thursday May 14, 2020

Try It! answer: 38

Answers: **7.** $\sum_{n=0}^{4} 5n$ **9.** $\sum_{k=1}^{5} 4$

Exercises for Friday, May 15, 2020

Try It! answer: 328
Answers: **31.** 49 **33.** 254 **35.** $S_7 = \frac{147}{2}$

Name:
Date:

Pre – Calculus Minor Assessment

1. Express the description of a sum using summation notation:

   The sum of $6k - 5$ from $k = -2$ to $k = 1$

2. State the indicated sum:

$$\sum_{n=1}^{9} 5 \cdot 2^{n-1}$$