GreatHearts®

Pre-Calculus: Week of May 18 – May 22

Time Allotment: 40 minutes per day

Student Name: ______________________________

Teacher Name: Mrs. Melisa R. Walters

Packet Overview

Date	Objective(s)	Page Number
Monday, May 18	Evaluate and Identify Functions	2
Tuesday, May 19	Calculating Sine, Cosine, and Tangent Using The Unit Circle	6
Wednesday, May 20	Solving Trigonometric Equations Algebraically by applying Trigonometric Identities.	11
Thursday, May 21	Examining Sequences	15
Friday, May 22	Minor Assessment	18

Zoom Guided Instruction:

Period 3 Monday and Wednesday from 1:00PM – 1:50 PM
Period 4 Tuesday and Thursday from 10:00AM – 10:50 AM
Period 6 Tuesday and Thursday from 1:00PM – 1:50 PM

Thank you for your hard work students. I appreciate all of you. Have a great day!

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

____

I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

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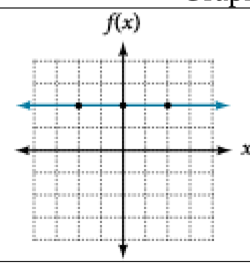
Monday, May 18

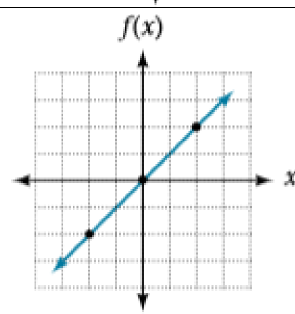
Pre-Calculus Review
Lesson: All About Functions

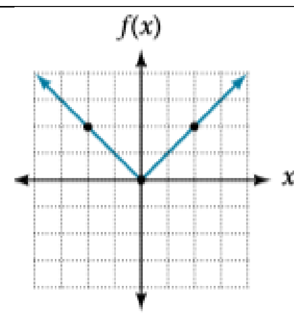
Objective: Evaluate and Identify Functions

Lesson 1
It is important to begin our review week with recalling functions. Remember that a function is a relationship between some variables. We have some set of input variables and an output variable. If our output variable is named f, and our input variables are x, y, z, … and so on, we use this notation: f(x, y, z, …) to indicate we have a function of our vars x, y, z and so on. The input or arguments are the variables x, y, z, … which are the independent variables. The value of f is the output or function value or dependent variable.

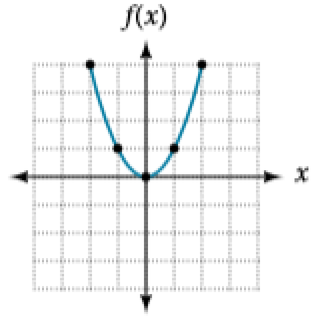
Let's focus on the functions from our toolkit functions that only involve one variable x. Real functions of one variable with one input variable x and output variable f, we use the notation f(x). *IMPORTANT to remember: f(x) does NOT mean f times x!*

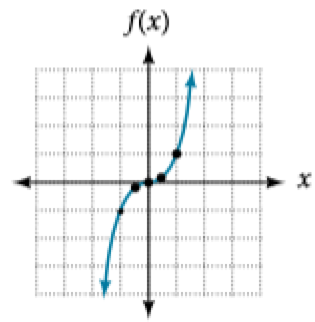
<table>
<tr><th colspan="3">Toolkit Functions</th></tr>
<tr><th>Name</th><th>Function</th><th>Graph</th></tr>
<tr><td>Constant</td><td>f (x) = c, where c is a constant</td><td>

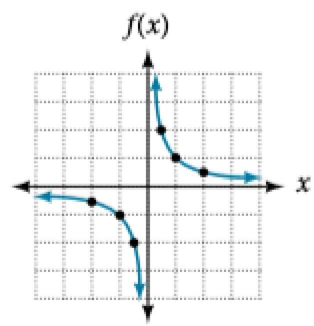
<table><tr><th>x</th><th>f(x)</th></tr><tr><td>–2</td><td>2</td></tr><tr><td>0</td><td>2</td></tr><tr><td>2</td><td>2</td></tr></table></td></tr>
<tr><td>Identity</td><td>f (x) = x</td><td>

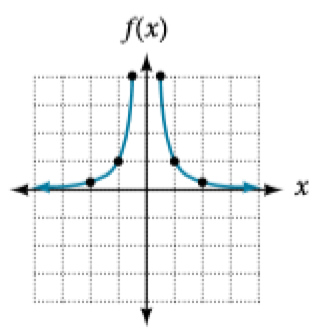
<table><tr><th>x</th><th>f(x)</th></tr><tr><td>–2</td><td>–2</td></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>2</td></tr></table></td></tr>
<tr><td>Absolute value</td><td>f (x) = | x |</td><td>

<table><tr><th>x</th><th>f(x)</th></tr><tr><td>–2</td><td>2</td></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>2</td></tr></table></td></tr>
</table>

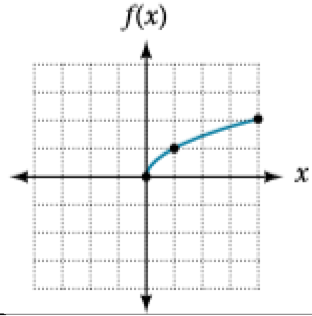
<table>
<tr>
<td>Quadratic</td>
<td>$f(x) = x^2$</td>
<td>


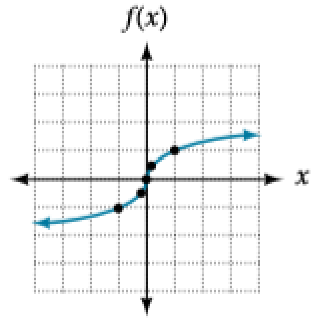
x	f(x)
–2	4
–1	1
0	0
1	1
2	4
</td>
</tr>
<tr>
<td>Cubic</td>
<td>$f(x) = x^3$</td>
<td>


x	f(x)
–1	–1
–0.5	–0.125
0	0
0.5	0.125
1	1
</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>$f(x) = \frac{1}{x}$</td>
<td>


x	f(x)
–2	–0.5
–1	–1
–0.5	–2
0.5	2
1	1
2	0.5
</td>
</tr>
<tr>
<td>Reciprocal squared</td>
<td>$f(x) = \frac{1}{x^2}$</td>
<td>


x	f(x)
–2	0.25
–1	1
–0.5	4
0.5	4
1	1
2	0.25
</td>
</tr>
</table>

<table>
<tr><td>Square root</td><td>$f(x) = \sqrt{x}$</td><td>

<table><tr><th>x</th><th>f(x)</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>4</td><td>2</td></tr></table></td></tr>
<tr><td>Cube root</td><td>$f(x) = \sqrt[3]{x}$</td><td>

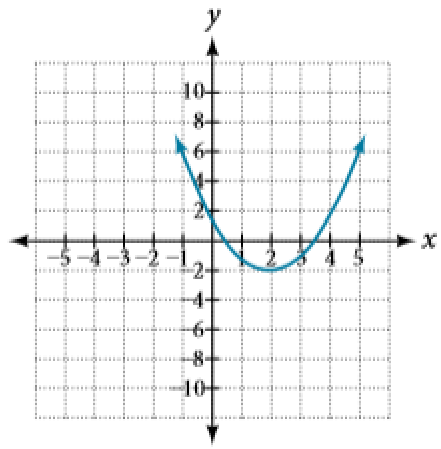
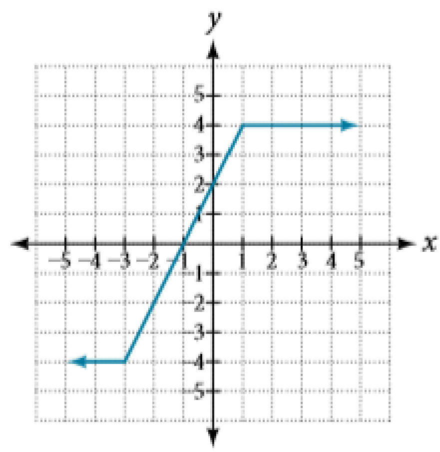
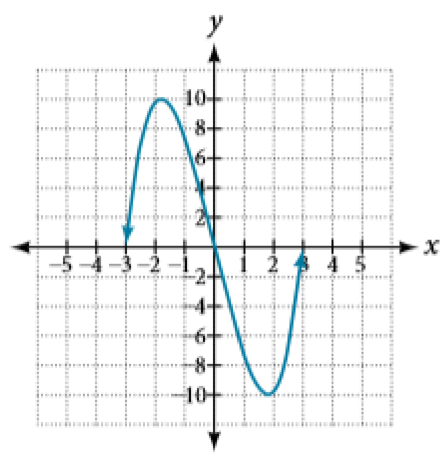
<table><tr><th>x</th><th>f(x)</th></tr><tr><td>–1</td><td>–1</td></tr><tr><td>–0.125</td><td>–0.5</td></tr><tr><td>0</td><td>0</td></tr><tr><td>0.125</td><td>0.5</td></tr><tr><td>1</td><td>1</td></tr></table></td></tr>
</table>

Review of Interval Notation

Inequality	Interval Notation	Graph on Number Line	Description
$x > a$	(a, ∞)		*x* is greater than *a*
$x < a$	$(-\infty, a)$		*x* is less than *a*
$x \geq a$	$[a, \infty)$		*x* is greater than or equal to *a*
$x \leq a$	$(-\infty, a]$		*x* is less than or equal to *a*
$a < x < b$	(a, b)		*x* is strictly between *a* and *b*
$a \leq x < b$	$[a, b)$		*x* is between *a* and *b*, to include *a*
$a < x \leq b$	$(a, b]$		*x* is between *a* and *b*, to include *b*
$a \leq x \leq b$	$[a, b]$		*x* is between *a* and *b*, to include *a* and *b*

Exercises for Monday, May 18, 2020

1. Evaluate the function $f(x) = -3x^2 + 2x$ at the given point f(-2).	2. Given f (x) = 2x 2 − 5x, find f (a + 1) − f (1)
For exercises 3 & 4, use the functions $f(x) = 3 - 2x^2 + x$ *and* $g(x) = \sqrt{x}$ to find the composite functions.	
3. $(g \circ f)(1)$	
Exercises 4 & 5, graph the functions by translating, stretching, and/or compressing a toolkit function.	
4. $f(x) = x^2 + 1$	5. $f(x) = (x+1)^2 + 1$

For the following exercises, use the graphs to determine the intervals on which the functions are increasing, decreasing, or constant.	
6. On what intervals is the function increasing? Decreasing?	
7. On what intervals is the function increasing? Constant?	
8. Find the absolute maximum of the function graphed.	

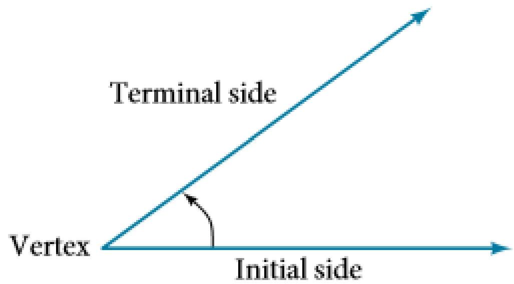
Tuesday, May 19

Pre-Calculus Review
Lesson 2: The Unit Circle

Objective: Calculating Sine, Cosine, and Tangent Using The Unit Circle

Lesson 2

Let's take a look at Trigonometric functions beyond the right angle. Consider the trigonometry in the unit circle. Suppose our angle has been moved into standard position.



If we take a unit circle centered at the origin, the terminal side intersects the unit circle.

In addition to knowing the measurements in degrees and radians of a quarter revolution, a half revolution, and a full revolution, there are other frequently encountered angles in one revolution of a circle with which we should be familiar. It is common to encounter multiples of 30, 45, 60, and 90 degrees. These values are shown in **Figure 14**. Memorizing these angles will be very useful as we study the properties associated with angles.

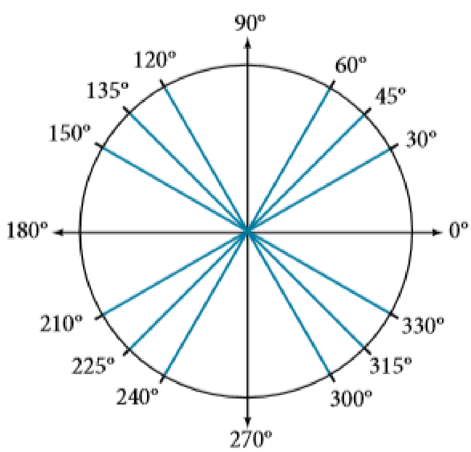


Figure 14 Commonly encountered angles measured in degrees

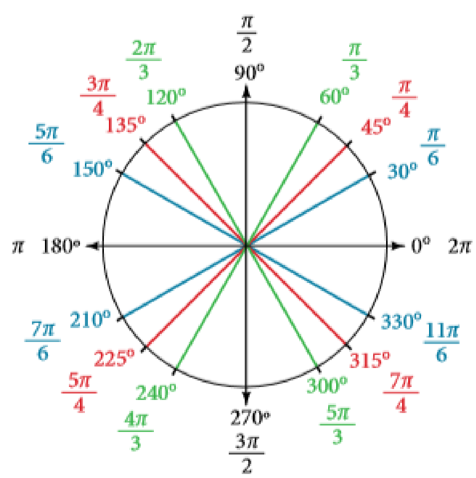


Figure 15 Commonly encountered angles measured in radians

Finding Function Values for the Sine and Cosine

To define our trigonometric functions, we begin by drawing a unit circle, a circle centered at the origin with radius 1, as shown in **Figure 2**. The angle (in radians) that t intercepts forms an arc of length s. Using the formula $s = rt$, and knowing that $r = 1$, we see that for a unit circle, $s = t$.

Recall that the x- and y-axes divide the coordinate plane into four quarters called quadrants. We label these quadrants to mimic the direction a positive angle would sweep. The four quadrants are labeled I, II, III, and IV.

For any angle t, we can label the intersection of the terminal side and the unit circle as by its coordinates, (x, y). The coordinates x and y will be the outputs of the trigonometric functions $f(t) = \cos t$ and $f(t) = \sin t$, respectively. This means $x = \cos t$ and $y = \sin t$.

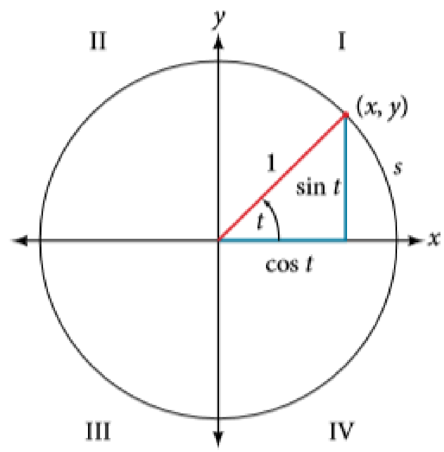


Figure 2 Unit circle where the central angle is t radians

We have now found the cosine and sine values for all of the most commonly encountered angles in the first quadrant of the unit circle. Table 1 summarizes these values.

Angle	0	$\frac{\pi}{6}$, or $30°$	$\frac{\pi}{4}$, or $45°$	$\frac{\pi}{3}$, or $60°$	$\frac{\pi}{2}$, or $90°$
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Table 1

Figure 14 shows the common angles in the first quadrant of the unit circle.

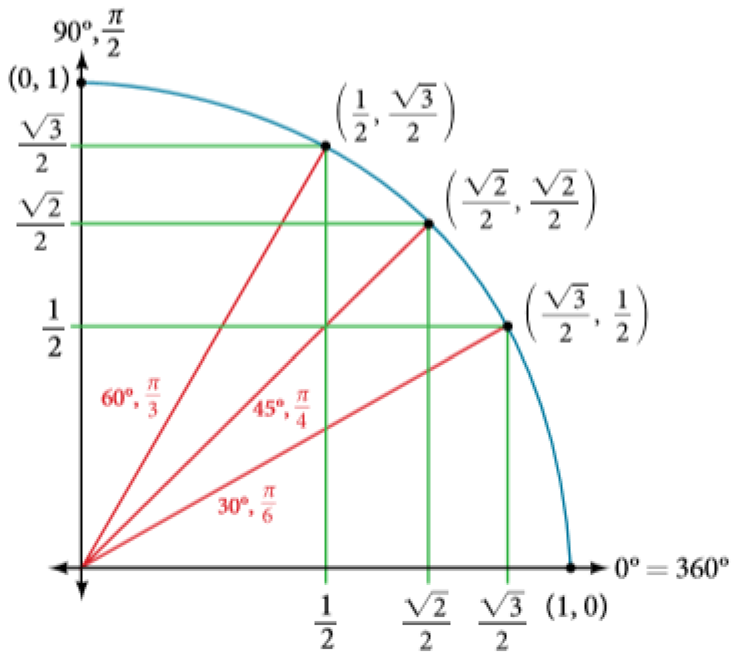


Figure 14

Now that we have learned how to find the cosine and sine values for special angles in the first quadrant, we can use symmetry and reference angles to fill in cosine and sine values for the rest of the special angles on the unit circle. They are shown in Figure 19. Take time to learn the (x, y) coordinates of all of the major angles in the first quadrant.

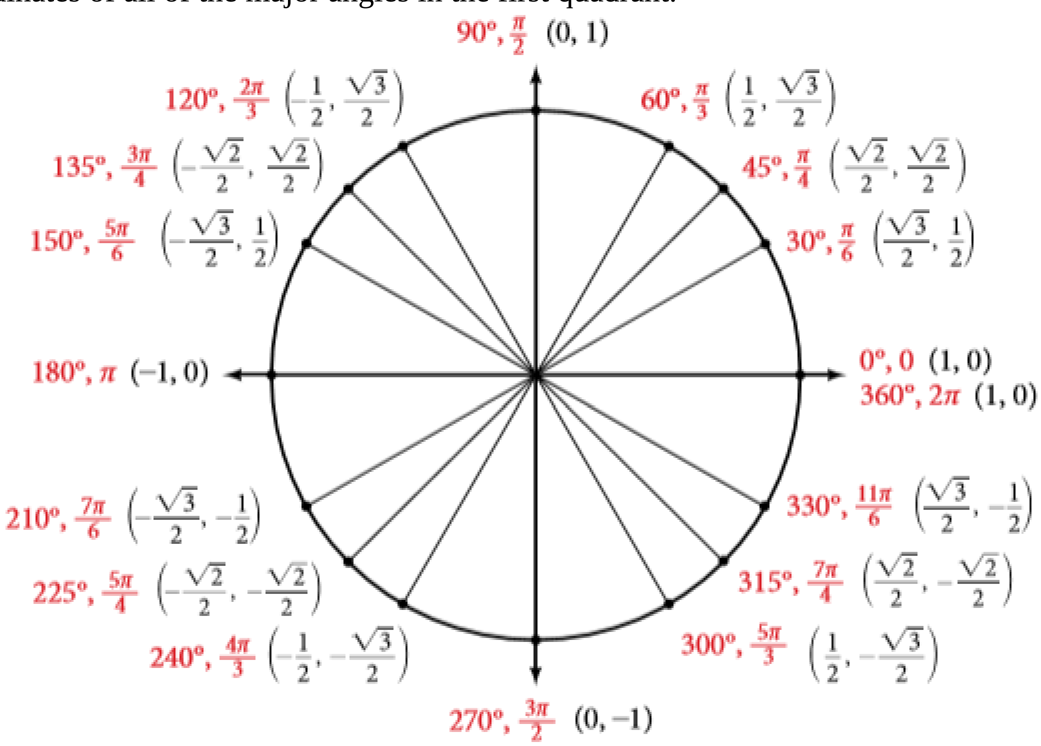


Figure 19 Special angles and coordinates of corresponding points on the unit circle

Exercises for Tuesday, May 19, 2020

1. Convert $\frac{5\pi}{6}$ radians to degrees.	2. Convert -620° to radians.
3. Draw the angle 315° in standard position on the Cartesian plane.	4. Draw the angle $-\frac{\pi}{4}$ in standard position on the Cartesian plane.
5. Find the exact value of $sin\,\frac{\pi}{6}$.	6. Compute sine of 240°.
7. State the range of the sine and cosine functions.	8. Find the exact value of $tan\,\frac{\pi}{3}$.
9. Find the exact value of sin 60°.	

Wednesday, May 20

Pre-Calculus Review
Lesson 3: Trigonometric Identities

Objective: Solving Trigonometric Equations Algebraically by applying Trigonometric Identities.

Lesson 3

Math is everywhere, even in places we might not immediately recognize. For example, mathematical relationships describe the transmission of images, light, and sound. The sinusoidal graph models music playing on a phone, radio, or computer. Such graphs are described using trigonometric equations and functions.

The trigonometric identities we examined can be traced to a Persian astronomer who lived around 950 AD, but the ancient Greeks discovered these same formulas much earlier and stated them in terms of chords. These are special equations or postulates, true for all values input to the equations, and with innumerable applications. The formulas that follow simplify many trigonometric expressions and equations. Keep in mind that the term formula is used synonymously with the word identity.

Pythagorean identities	$\sin^2\theta + \cos^2\theta = 1$ $1 + \cot^2\theta = \csc^2\theta$ $1 + \tan^2\theta = \sec^2\theta$
Even-odd identities	$\tan(-\theta) = -\tan\theta$ $\cot(-\theta) = -\cot\theta$ $\sin(-\theta) = -\sin\theta$ $\csc(-\theta) = -\csc\theta$ $\cos(-\theta) = \cos\theta$ $\sec(-\theta) = \sec\theta$
Reciprocal identities	$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient identities	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
Sum Formula for Cosine	$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
Difference Formula for Cosine	$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
Sum Formula for Sine	$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
Difference Formula for Sine	$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
Sum Formula for Tangent	$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
Difference Formula for Tangent	$\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$
Cofunction identities	$\sin\theta = \cos\left(\dfrac{\pi}{2}-\theta\right)$
	$\cos\theta = \sin\left(\dfrac{\pi}{2}-\theta\right)$
	$\tan\theta = \cot\left(\dfrac{\pi}{2}-\theta\right)$
	$\cot\theta = \tan\left(\dfrac{\pi}{2}-\theta\right)$
	$\sec\theta = \csc\left(\dfrac{\pi}{2}-\theta\right)$
	$\csc\theta = \sec\left(\dfrac{\pi}{2}-\theta\right)$
Double-angle formulas	$\sin(2\theta) = 2\sin\theta\cos\theta$
	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
	$= 1 - 2\sin^2\theta$
	$= 2\cos^2\theta - 1$
	$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$
Reduction formulas	$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$
	$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$
	$\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Half-angle formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$$

Product-to-sum Formulas

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$$

Sum-to-product Formulas

$$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Exercise for Wednesday, May 20, 2020

1. Simplify the given expression: $\cos(-x)\, sinxcotx + sin^2x$

2. Find the exact value: $cos\frac{7\pi}{12}$

3. Find the exact solution to the equation on [0, 2π).
   $\cos(2x) + sin^2x$

4. Prove the Identity: $tan^3x - tanxsec^2x = \tan(-x)$

Thursday, May 21

Pre-Calculus Review
Lesson 4: Sequences

Objective: Examining Sequences

Lesson 4:

sequence

A **sequence** is a function whose domain is the set of positive integers. A **finite sequence** is a sequence whose domain consists of only the first n positive integers. The numbers in a sequence are called **terms**. The variable a with a number subscript is used to represent the terms in a sequence and to indicate the position of the term in the sequence.

$$a_1, a_2, a_3, \ldots, a_n, \ldots$$

We call a_1 the first term of the sequence, a_2 the second term of the sequence, a_3 the third term of the sequence, and so on. The term an is called the ***nth* term of the sequence,** or the general term of the sequence. An **explicit formula** defines the nth term of a sequence using the position of the term. A sequence that continues indefinitely is an **infinite sequence**.

recursive formula

A **recursive formula** is a formula that defines each term of a sequence using preceding term(s). Recursive formulas must always state the initial term, or terms, of the sequence.

arithmetic sequence

An **arithmetic sequence** is a sequence that has the property that the difference between any two consecutive terms is a constant. This constant is called the **common difference**. If a_1 is the first term of an arithmetic sequence and d is the common difference, the sequence will be:

$$\{a_n\} = \{a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots\}$$

recursive formula for an arithmetic sequence

The recursive formula for an arithmetic sequence with common difference d is:

$$a_n = a_{n-1} + d \quad n \geq 2$$

explicit formula for an arithmetic sequence

An explicit formula for the nth term of an arithmetic sequence is given by

$$a_n = a_1 + d\,(n - 1)$$

definition of a geometric sequence

A **geometric sequence** is one in which any term divided by the previous term is a constant. This constant is called the **common ratio** of the sequence. The common ratio can be found by dividing any term in the sequence by the previous term. If a_1 is the initial term of a geometric sequence and r is the common ratio, the sequence will be

$$\{a_1, a_1r, a_1r^2, a_1r^3, \ldots\}.$$

recursive formula for a geometric sequence
The recursive formula for a geometric sequence with common ratio r and first term a_1 is

$$a_n = ra_{n-1}, n \geq 2$$

explicit formula for a geometric sequence
The nth term of a geometric sequence is given by the explicit formula:

$$a_n = a_1 r^{n-1}$$

summation notation
The sum of the first n terms of a **series** can be expressed in **summation notation** as follows:

$$\sum_{k=1}^{n} a_k$$

This notation tells us to find the sum of a_k from $k = 1$ to $k = n$.

k is called the **index of summation**, 1 is the **lower limit of summation**, and n is the **upper limit of summation**.

formula for the sum of the first n terms of a geometric series
A **geometric series** is the sum of the terms in a geometric sequence. The formula for the sum of the first n terms of a geometric sequence is represented as

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad r \neq 1$$

Any questions right now? Please email me at melisa.walters@greatheartsnorthernoaks.org

Exercises for Thursday May 20, 2020
Chapter 11 Test page 1015 #1, 3, 11, 13

1. Write the first four terms of the sequence defined by the recursive formula $a = -14$, $a_n = \dfrac{2 + a_{n-1}}{2}$.

2. Is the sequence 0.3, 1.2, 2.1, 3, … arithmetic? If so find the common difference.
3. Use summation notation to write the sum of terms $3k^2 - \frac{5}{6}k$ from $k = -3$ to $k = 15$.
4. Use the formula for the sum of the first n terms of a geometric series to find $\sum_{k=1}^{7} -0.2 \cdot (-5)^{k-1}$.

Friday, May 22

Pre-Calculus Review
Lesson: Minor Assessment

Please put your answers in the space provided. Then, upload this Minor Assessment onto Google classroom under "Minor Assessment Submission".

1. Evaluate the function $f(x) = -5x^2 + x$ at the given point f(3).	2. Given $f(x) = 2x^2 - 5$, find f (a + 1) − f (1)
For exercises 3 & 4, use the functions $\boldsymbol{f(x) = 3 - 2x^2 + x}$ *and* $\boldsymbol{g(x) = \sqrt{x}}$ to find the composite functions.	
3. $(g \circ f)(2)$	4. $(f \circ g)(1)$
Exercises 5 & 6, graph the functions by translating, stretching, and/or compressing a toolkit function.	
5. $f(x) = \sqrt{x+6} - 1$	6. $f(x) = \frac{1}{x+2} - 1$

For the following exercises, use the graph of g shown in Figure 1.

7. On what intervals is the function increasing?

8. On what intervals is the function decreasing?

9. Approximate the local minimum of the function. Express the answer as an ordered pair.

10. Approximate the local maximum of the function. Express the answer as an ordered pair.

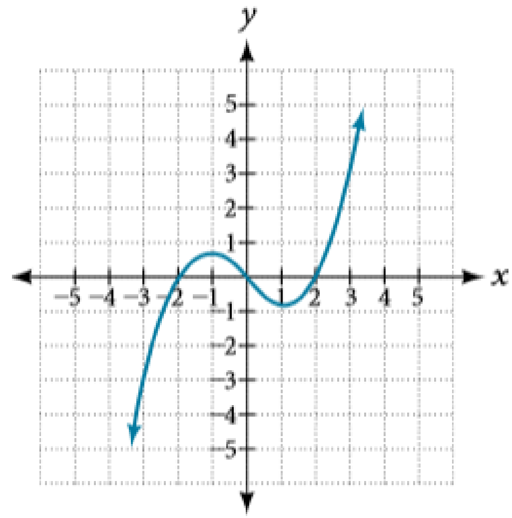


Figure 1

11. Convert $\frac{4\pi}{3}$ radians to degrees.

12. Convert 720° to radians.

13. Draw the angle 240° in standard position on the Cartesian plane.

14. Draw the angle $-\frac{\pi}{3}$ in standard position on the Cartesian plane.

15. Find the exact value of $sin\,\frac{\pi}{3}$.

16. Compute sine of 120°.

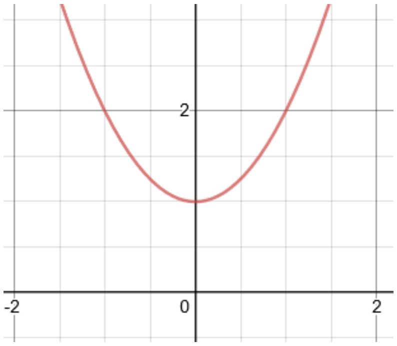
17. Find the exact value of sin 60°.

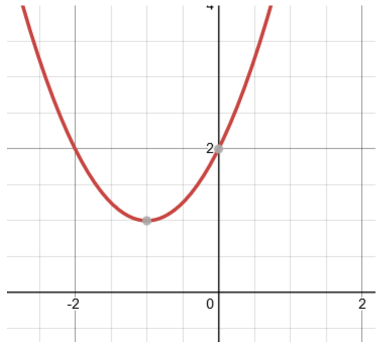
18. Use summation notation to write the sum of terms $\frac{1}{2}m + 5\ from\ m = 0\ to\ m = 5$.

ANSWER KEY

<u>Exercises for Monday, May 18, 2020</u>

1. -16
2. $2a^2$ – a
3. $\sqrt{2}$

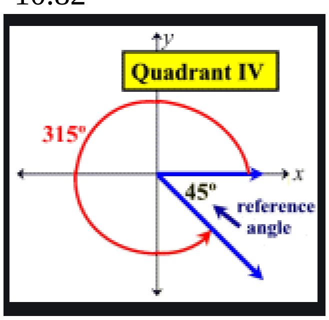
4. 

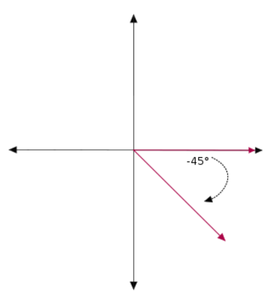
5. 

6. Increasing on (2, ∞), decreasing on (−∞, 2)
7. Increasing on (−3, 1), constant on (−∞, −3) and (1, ∞)
8. Absolute maximum: 10

<u>Exercises for Tuesday May 19, 2020</u>

1. 150°
2. -10.82

3. 

4. 

5. $\frac{1}{2}$ 6. $-\frac{\sqrt{3}}{2}$ 7. [-1,1] 8. $\sqrt{3}$ 9. $\frac{\sqrt{3}}{2}$

<u>Exercises for Wednesday May 20, 2020</u>

`1. 1 2. $\frac{\sqrt{2}-\sqrt{6}}{4}$ 3. $\frac{\pi}{2}, \frac{3\pi}{2}$

4.

$$\begin{aligned}\tan^3 x - \tan x \sec^2 x &= \tan x\,(\tan^2 x - \sec^2 x)\\ &= \tan x\,(\tan^2 x - (1 + \tan^2 x))\\ &= \tan x\,(\tan^2 x - 1 - \tan^2 x)\\ &= -\tan x = \tan(-x) = \tan(-x)\end{aligned}$$

<u>Answers to Exercises for Thursday May 21, 2020</u>

1. $-14, -6, -2, 0$
2. The sequence is arithmetic. The common difference is $d = 0.9$
3. $$\sum_{k=-3}^{15}\left(3k^2 - \frac{5}{6}k\right)$$
4. $S_7 = -2{,}604.2$